The complex interplay between structure and function in the brain: from experiments to theory.

a. Pinwheel-Dipole Topology of the cat visual cortex

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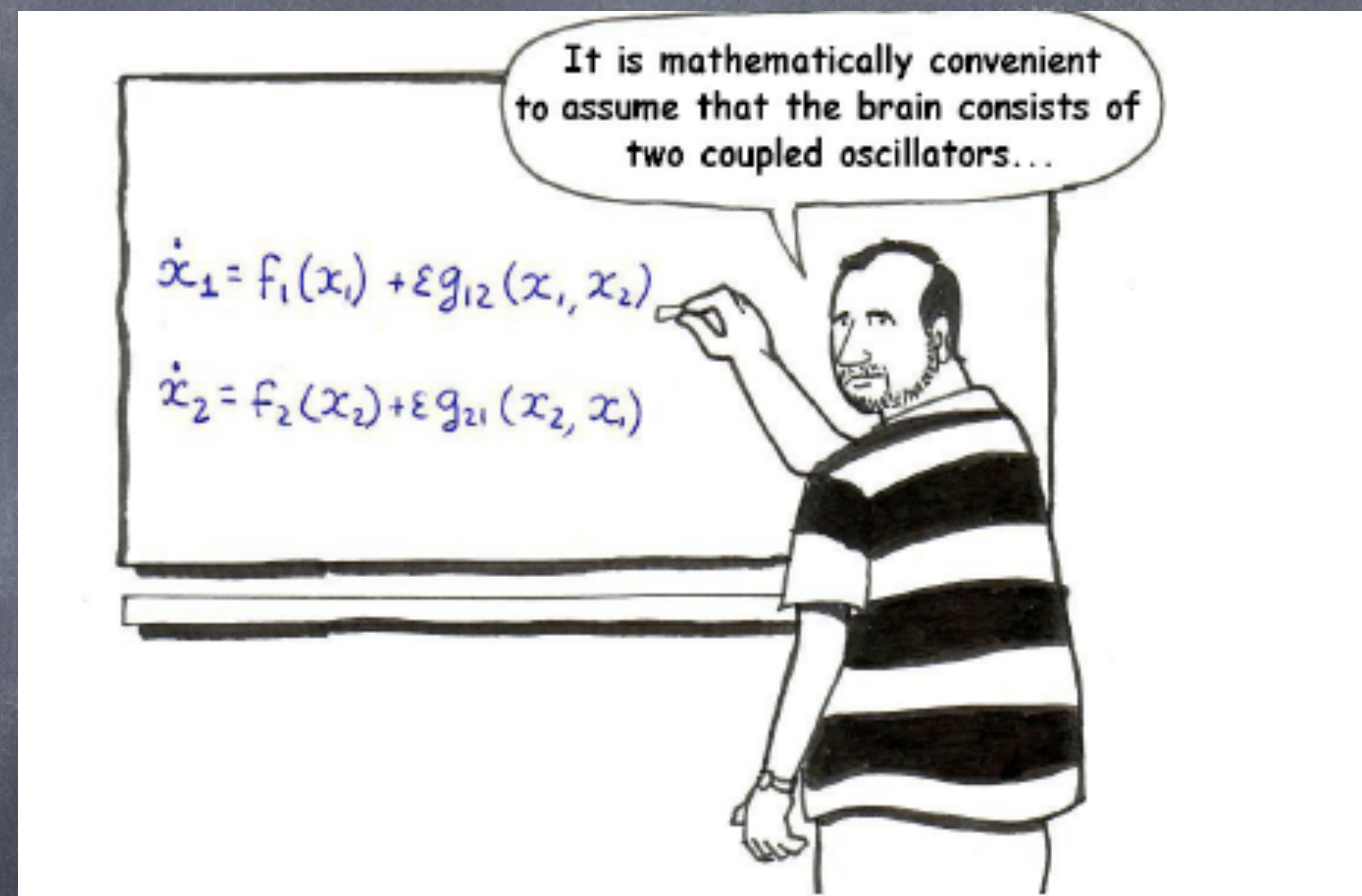
Center for Interdisciplinary Research in Biology
&
Mycenae Team

Between Theory and Applications: Mathematics in Action - Bedlewo - May 2015

Collaborators (a disclaimer)

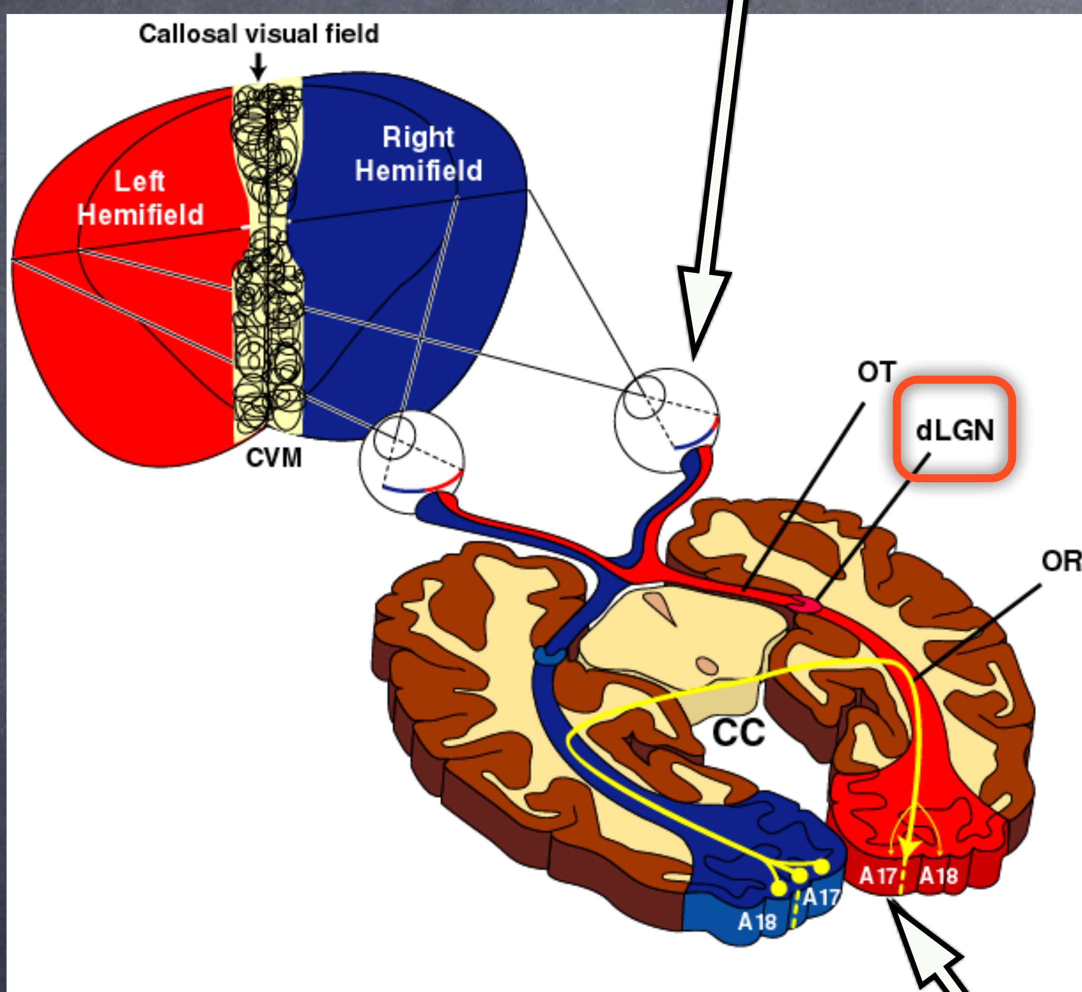
J. Ribot (postdoc @ Mathematical Neuroscience Team), experimentalist
A. Romagnoni (postdoc @ Mathematical Neuroscience Team), physicist
D. Bennequin and C. Milleret (experimental setup)



How concurrent representation of sensory features is organized in neocortex?

The visual system organisation

Retina



Primary Visual Cortex V1 (A17)

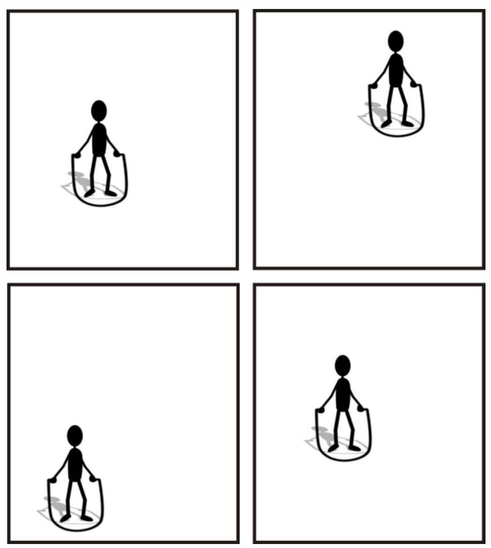
« Where »
Position, Depth, Motion…

V1

« What »
Shape, Color, Texture…

Attributes of the visual scene

Location



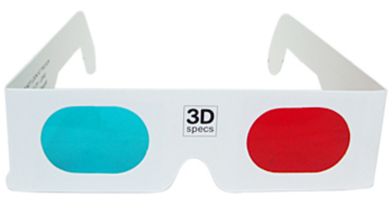
Shape

Motion

Details (Spatial frequency)

Colors

Depth



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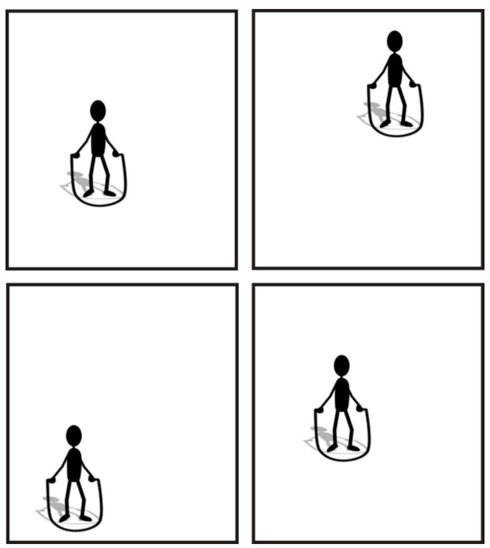

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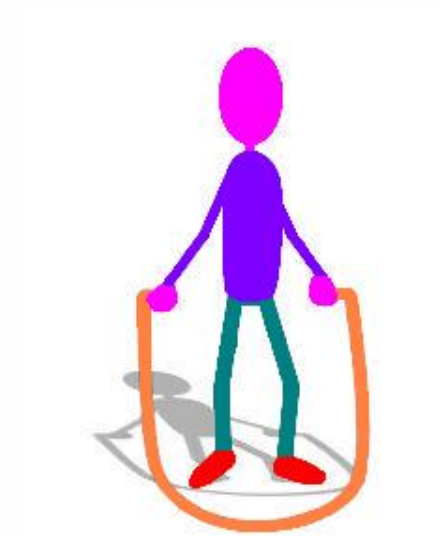
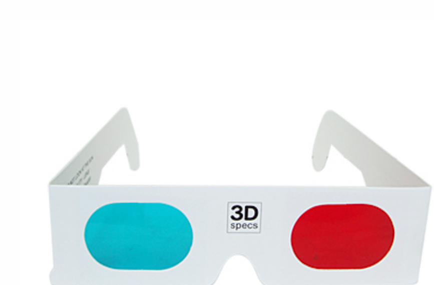


Depth



Local selectivity in V1

- **Electrophysiology**

  Hubel & Wiesel 1962

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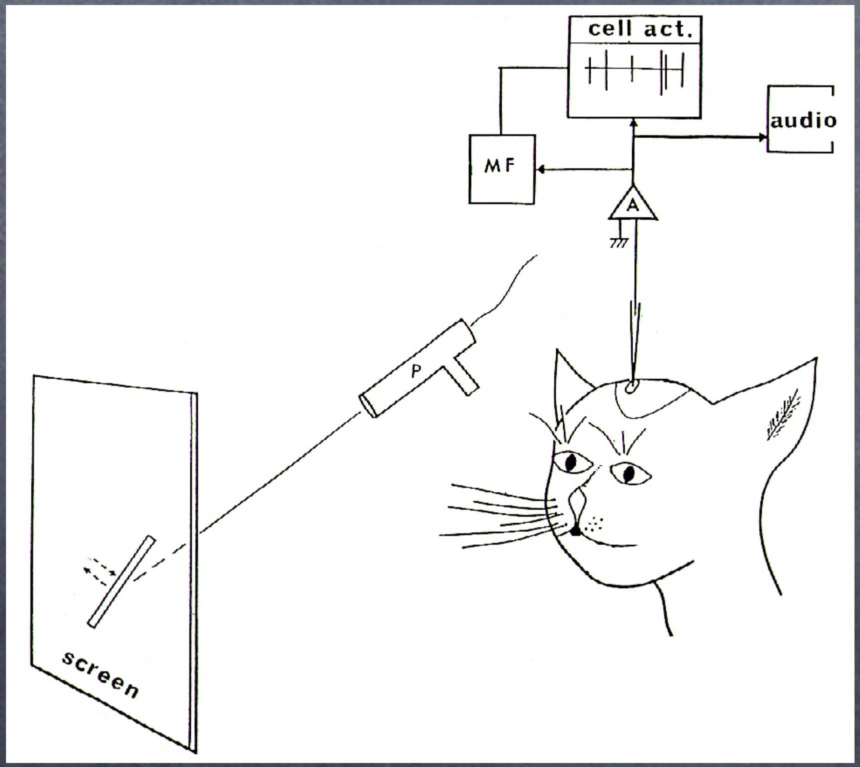

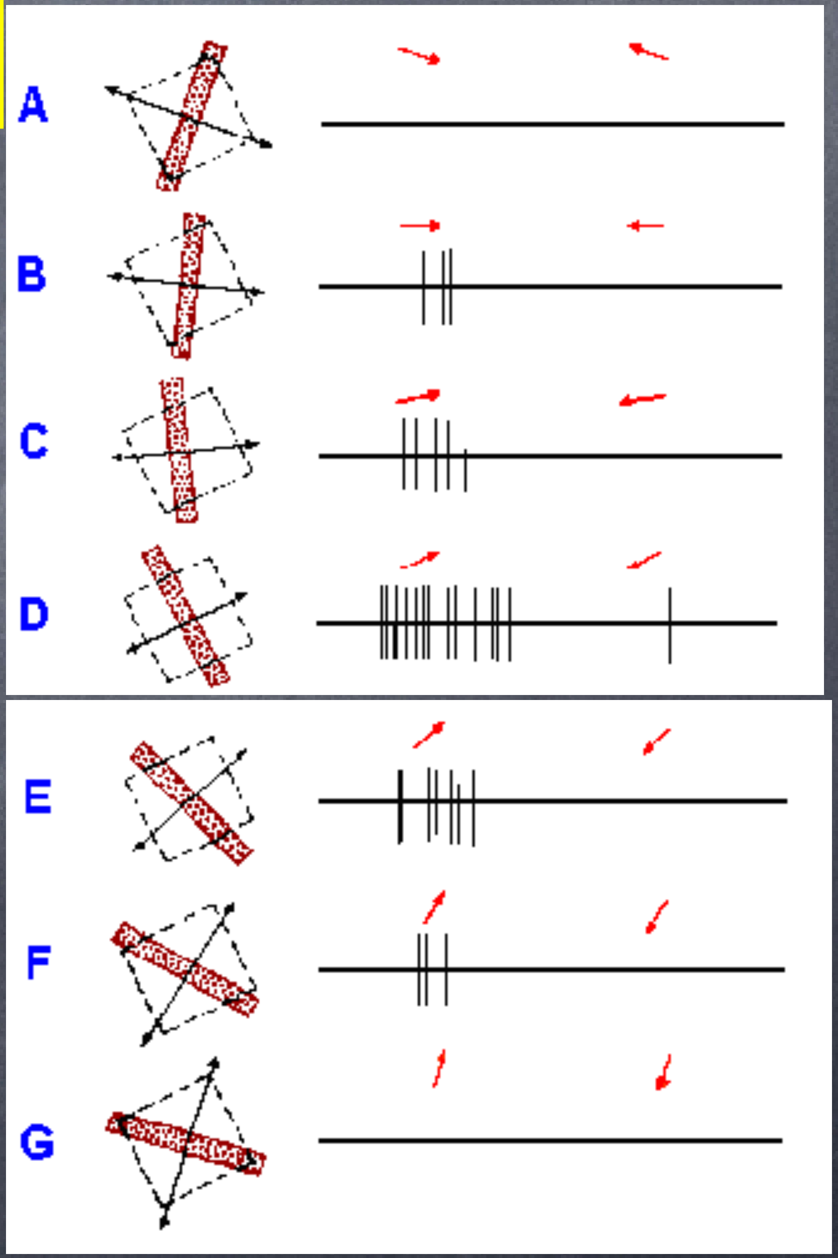

Local selectivity in V1

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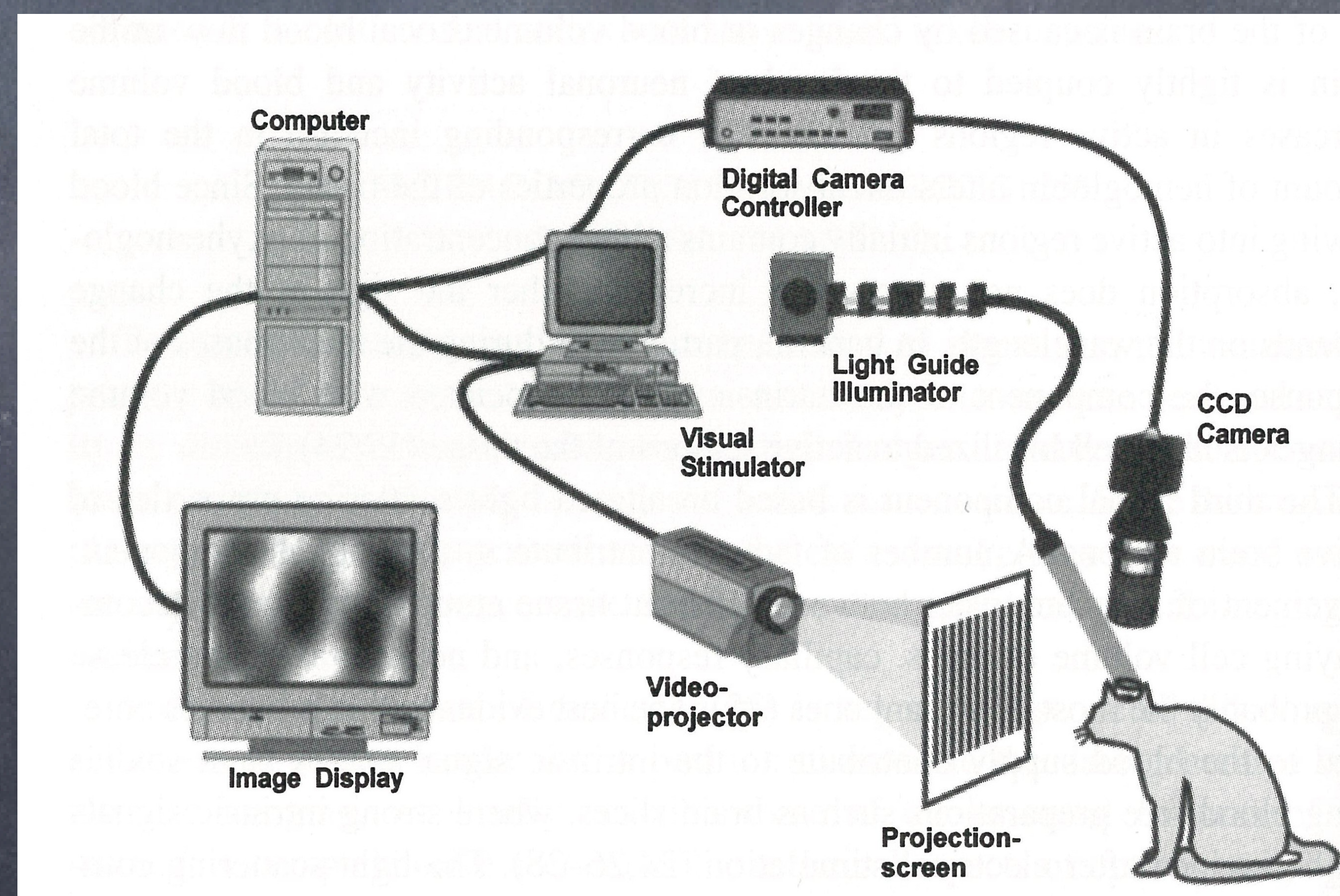
  Hubel & Wiesel 1962

- **Optical imaging**

  Bonhöffer & Grinvald 1991



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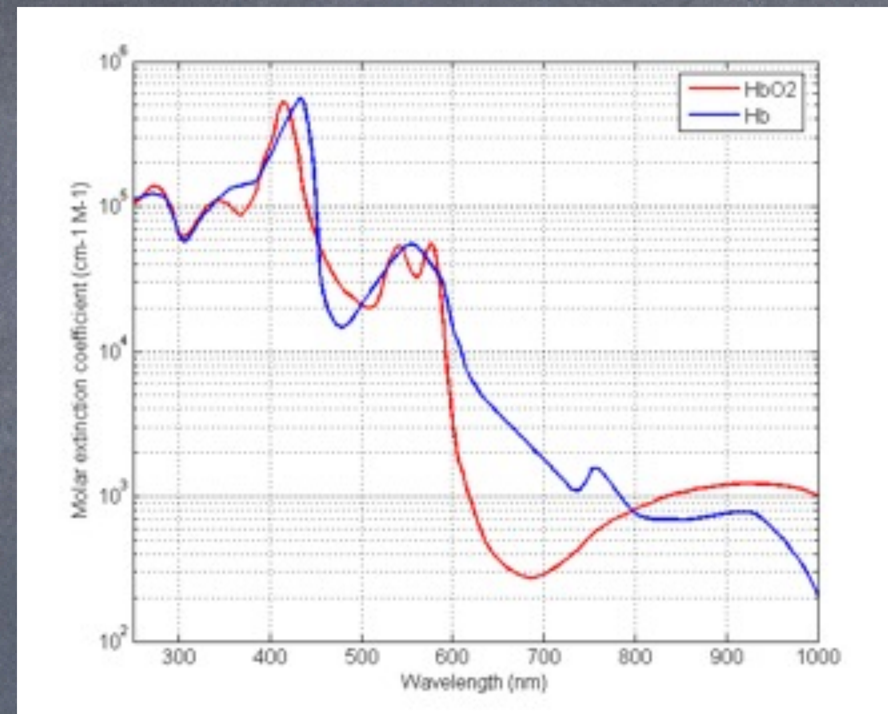

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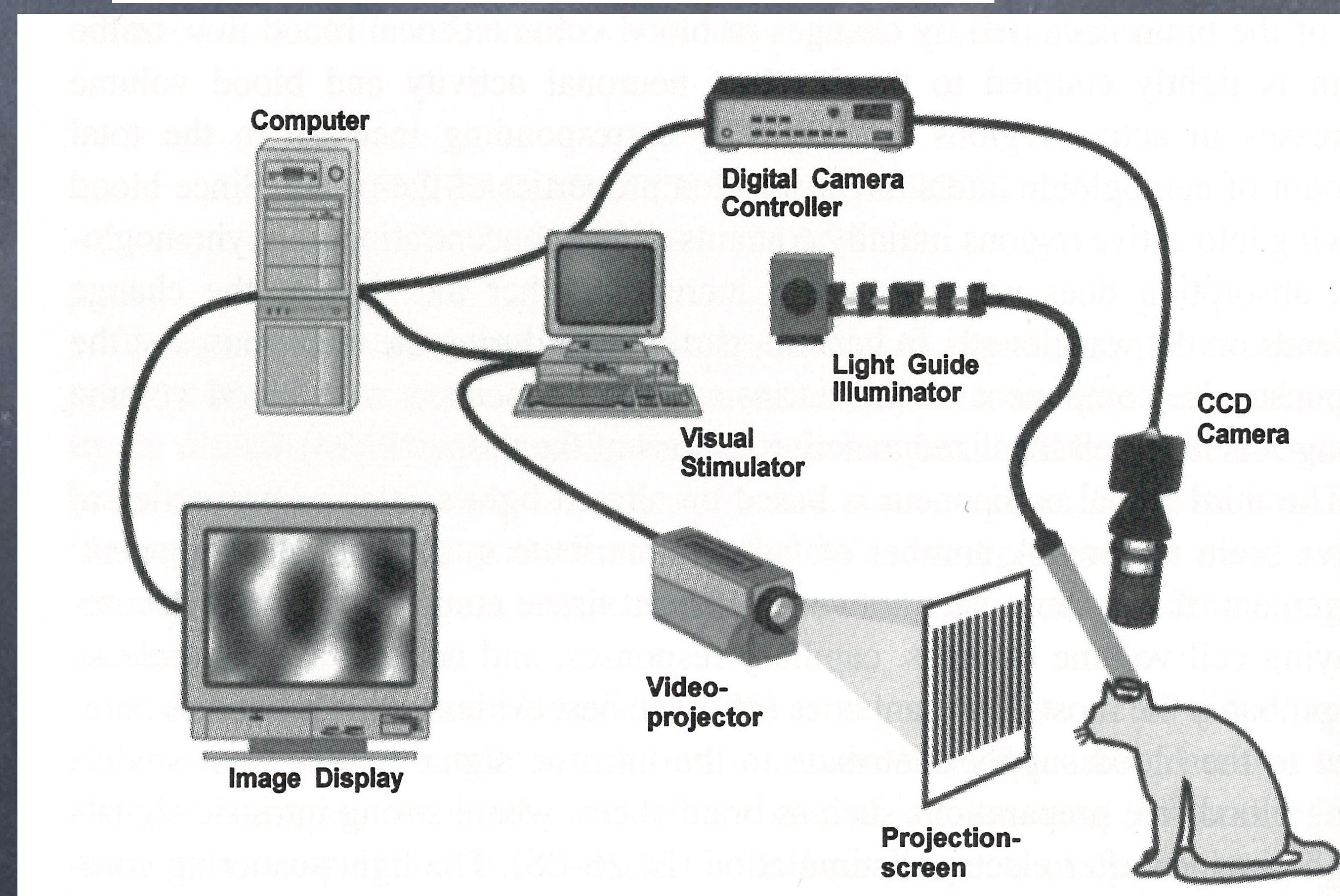
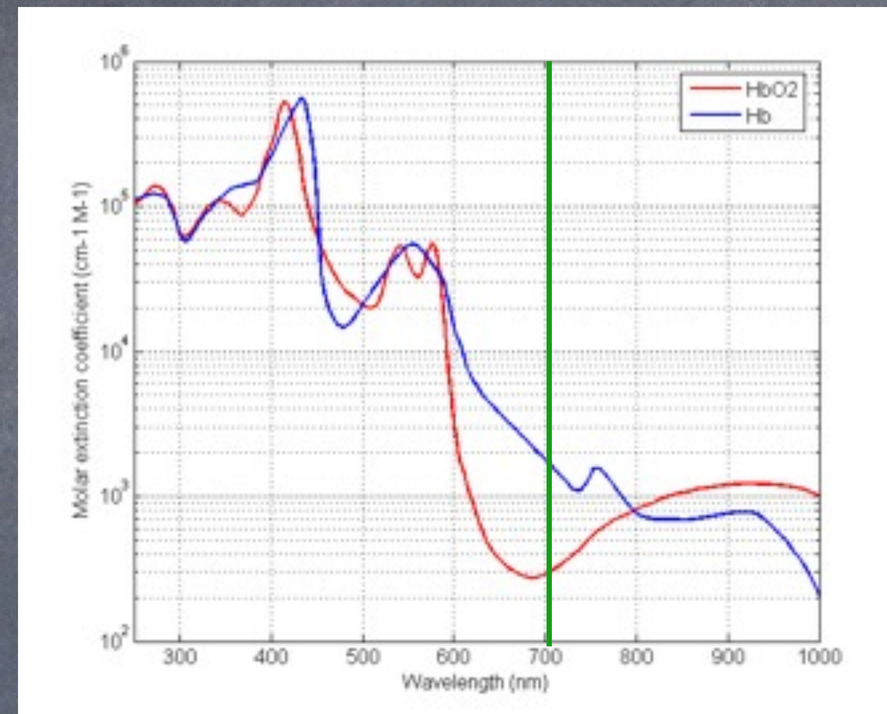


Local selectivity in V1



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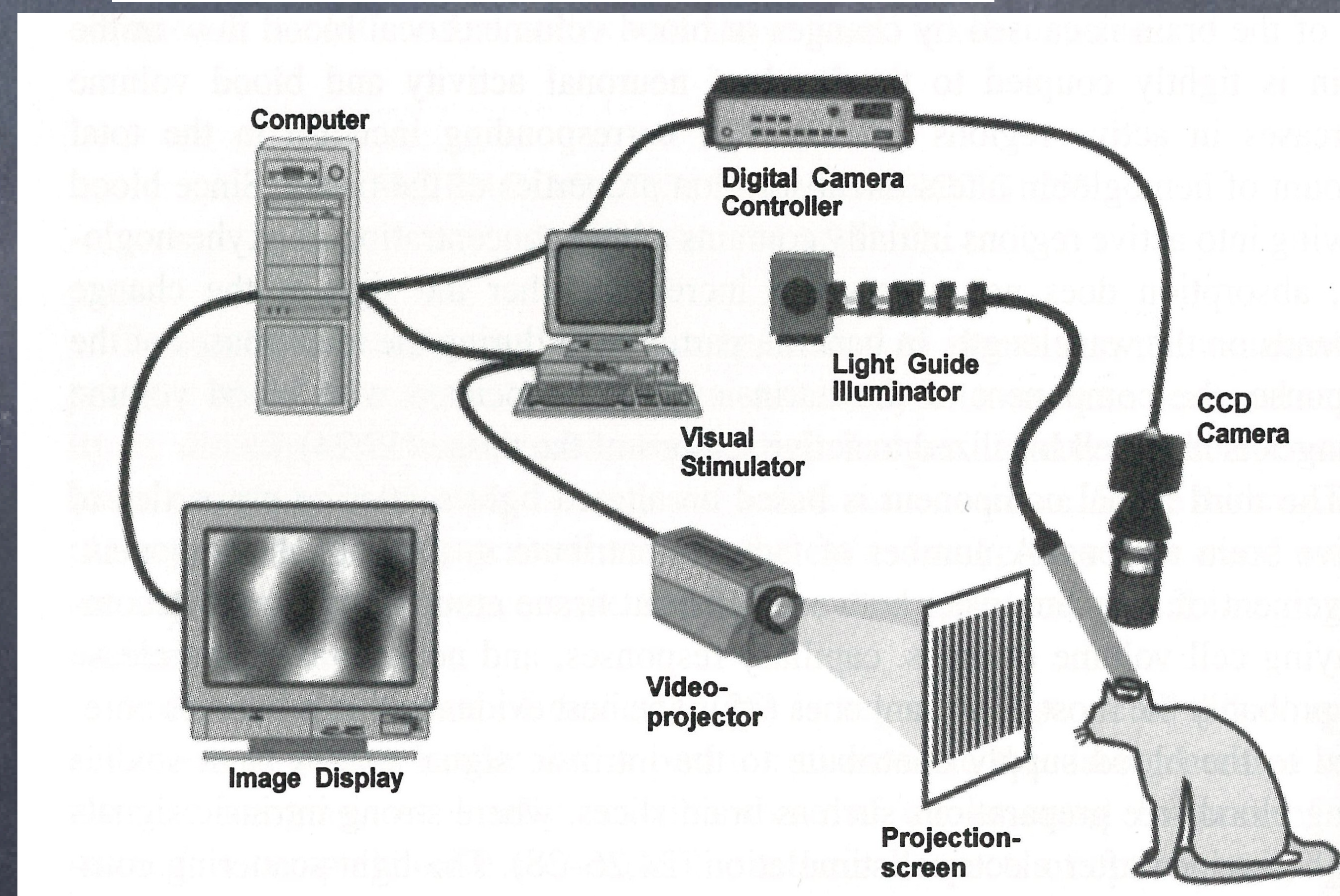
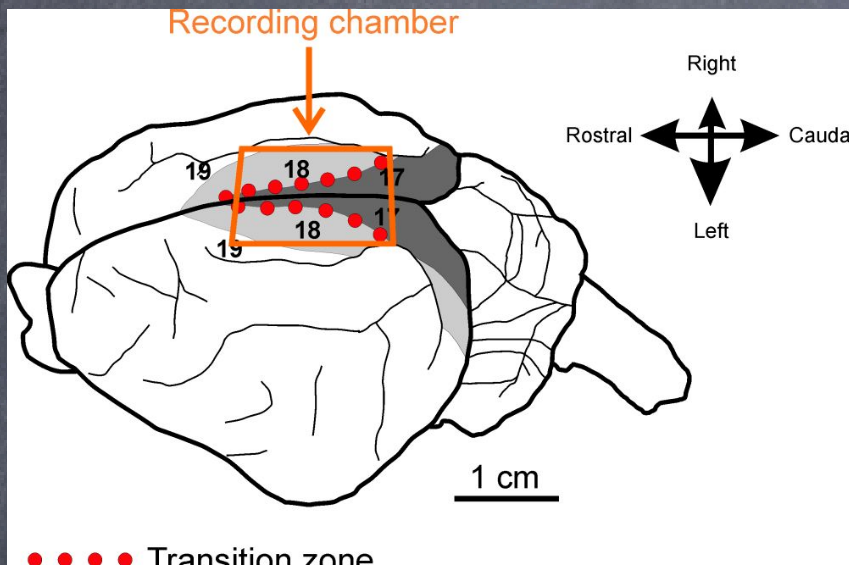
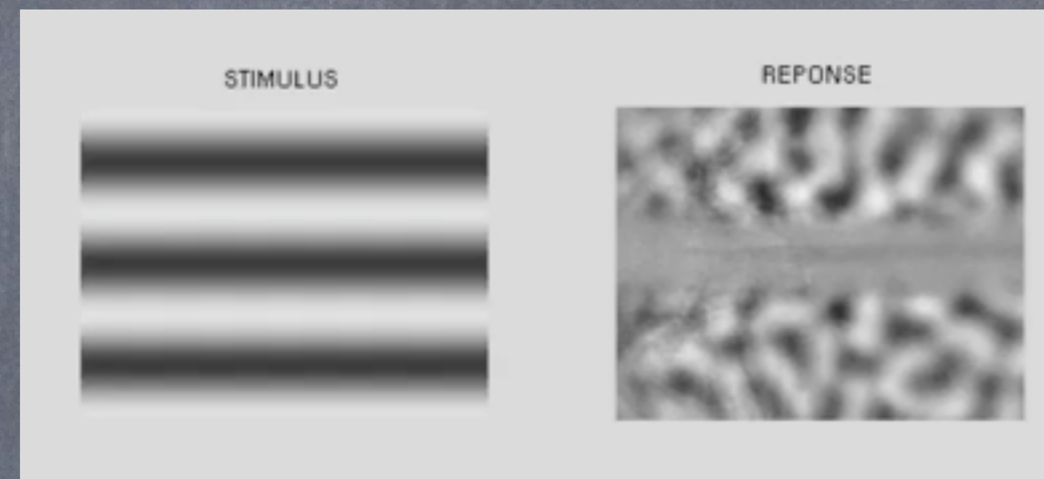
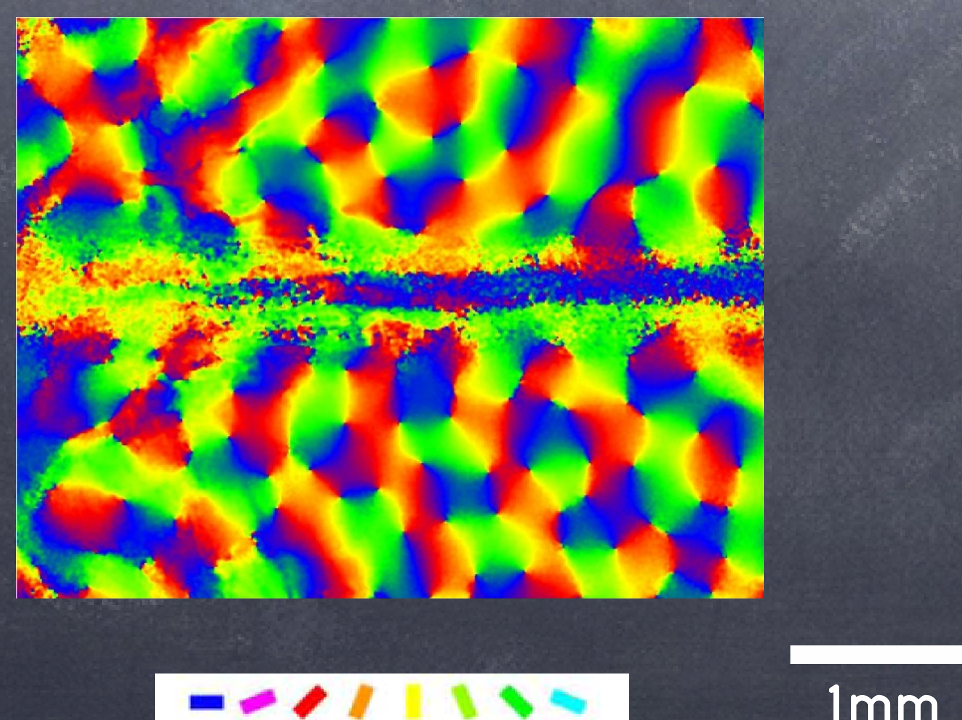


The example of the orientation map

Cat visual cortex







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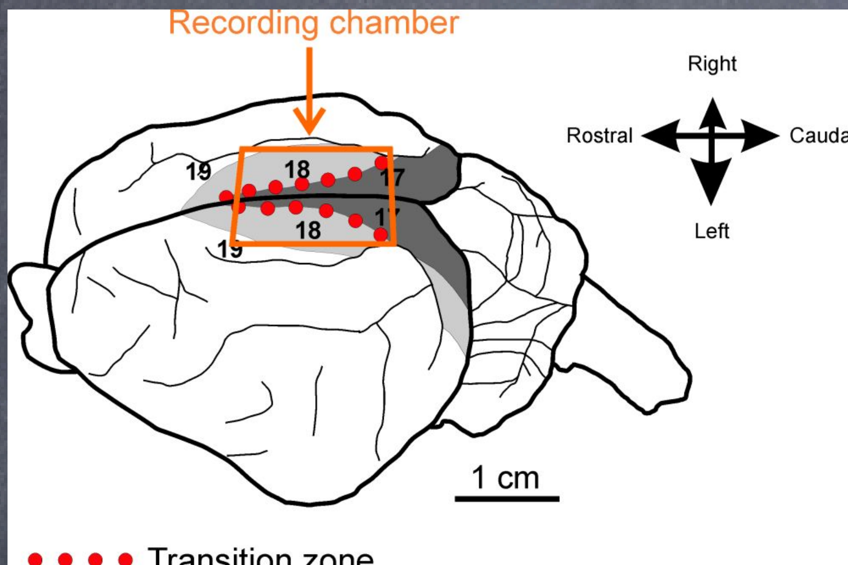
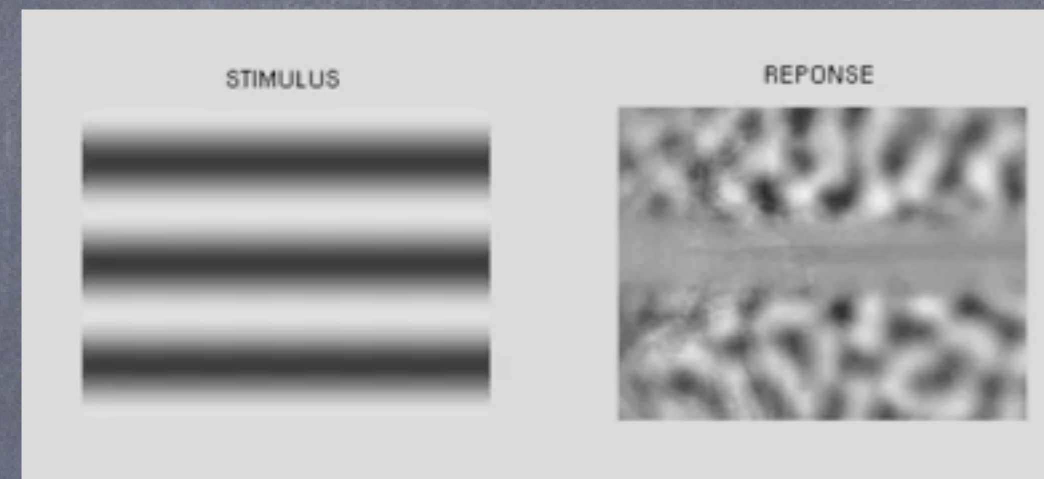
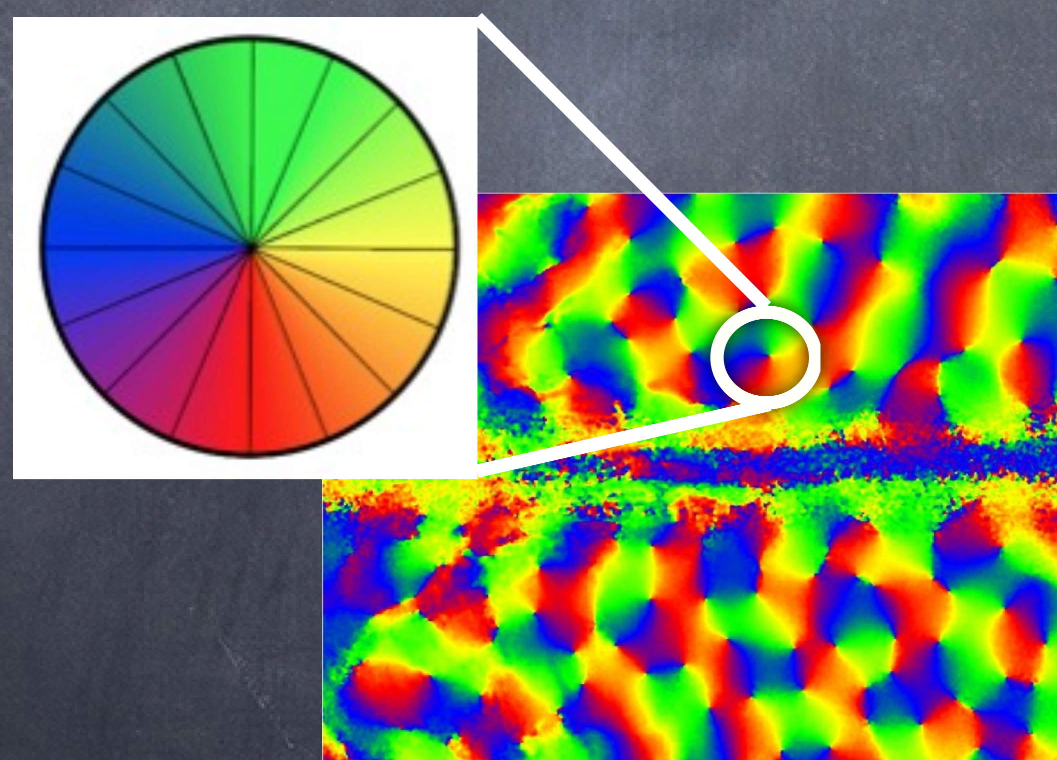
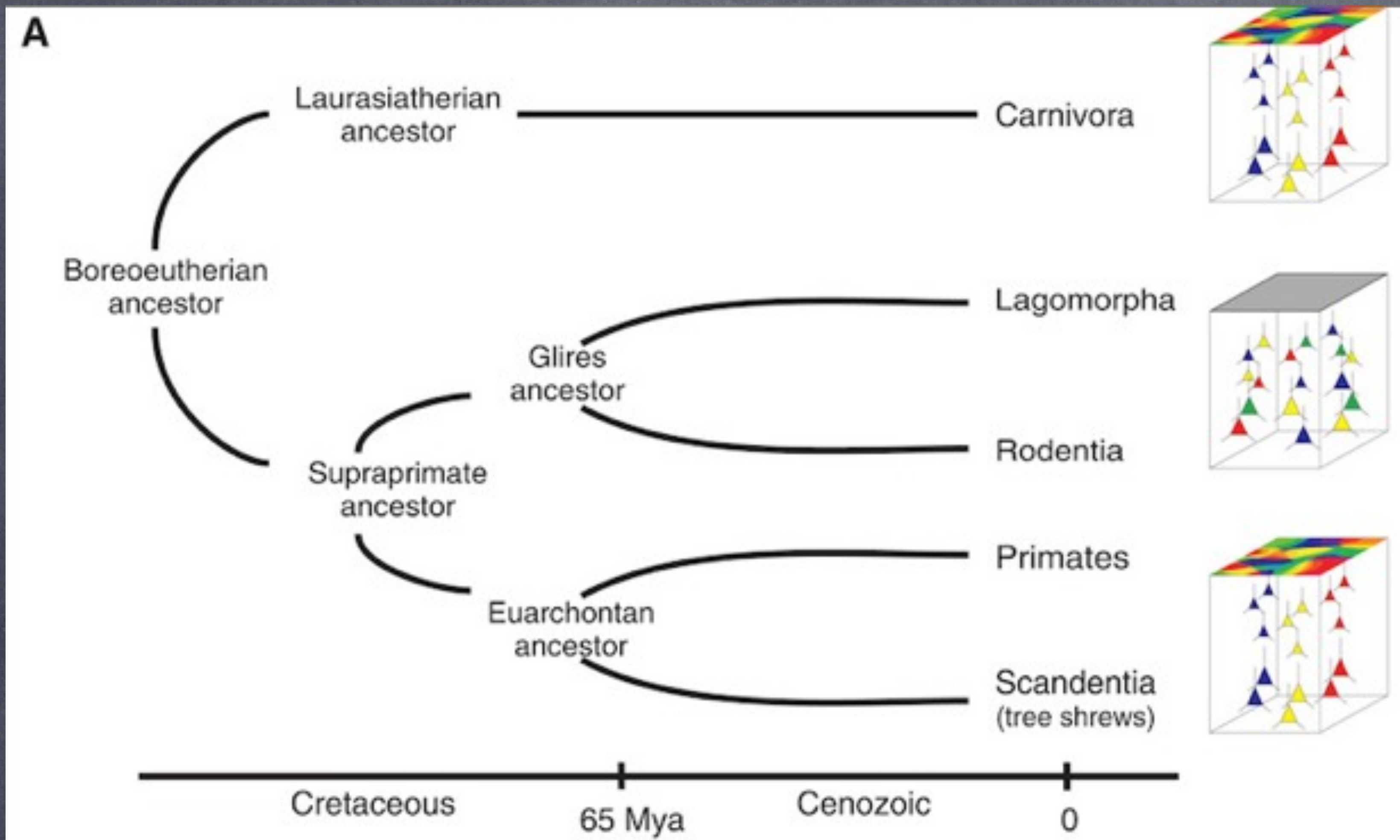




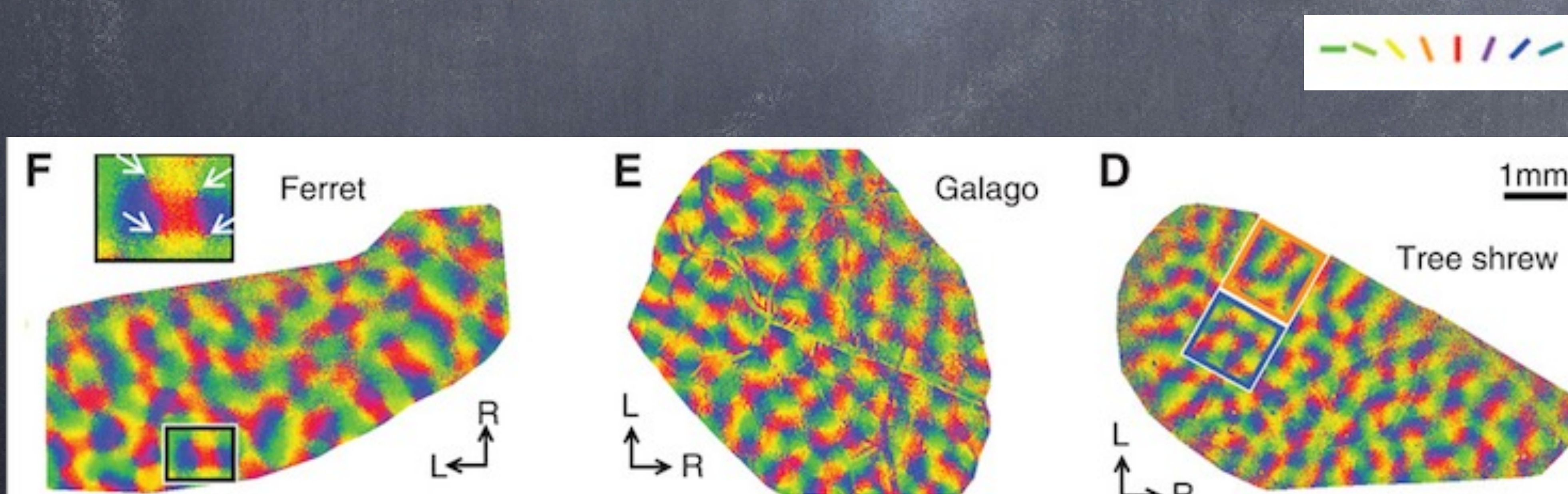


1mm

Universality of the map

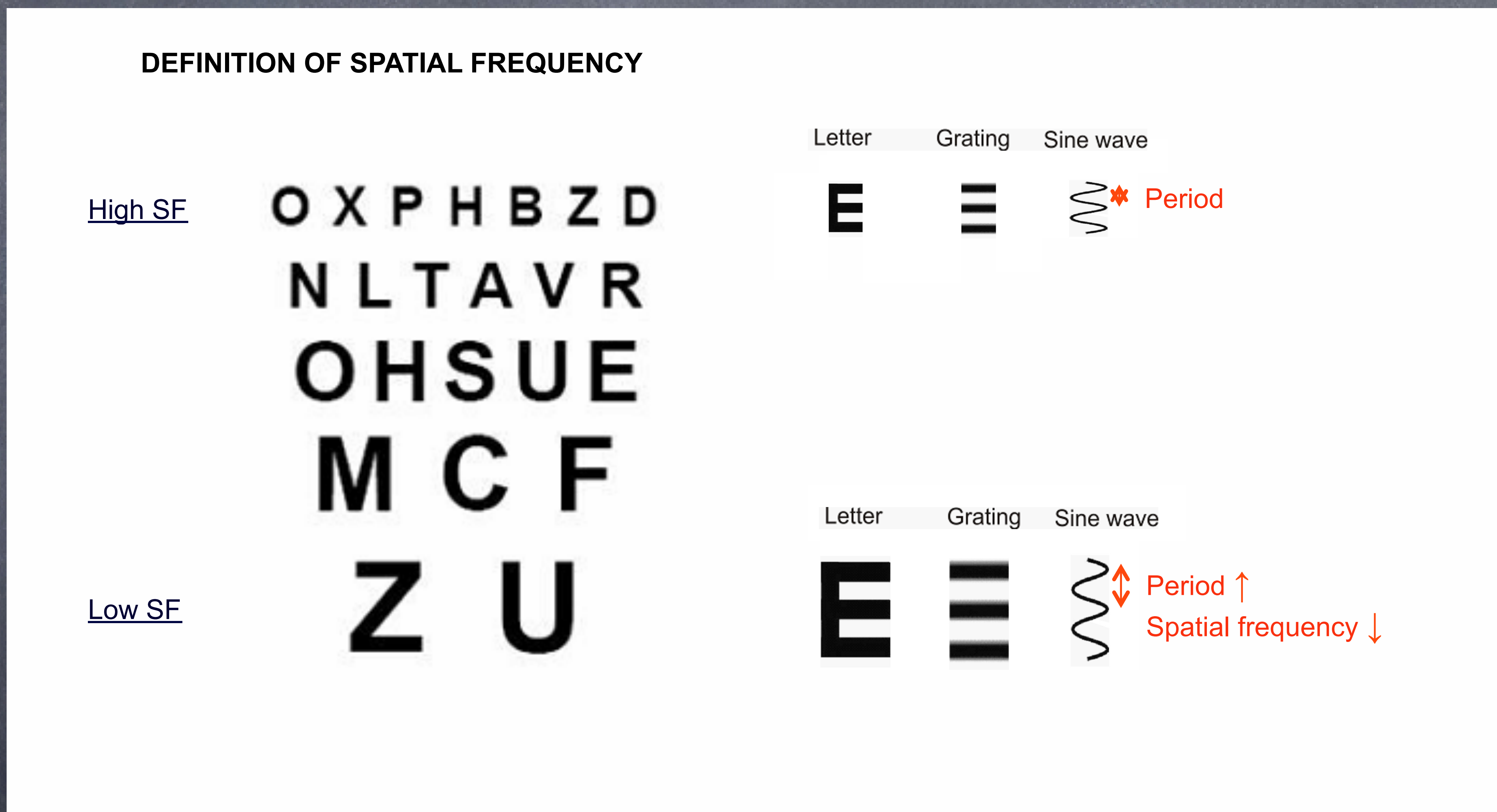


Kaschube et al, Science (2010)



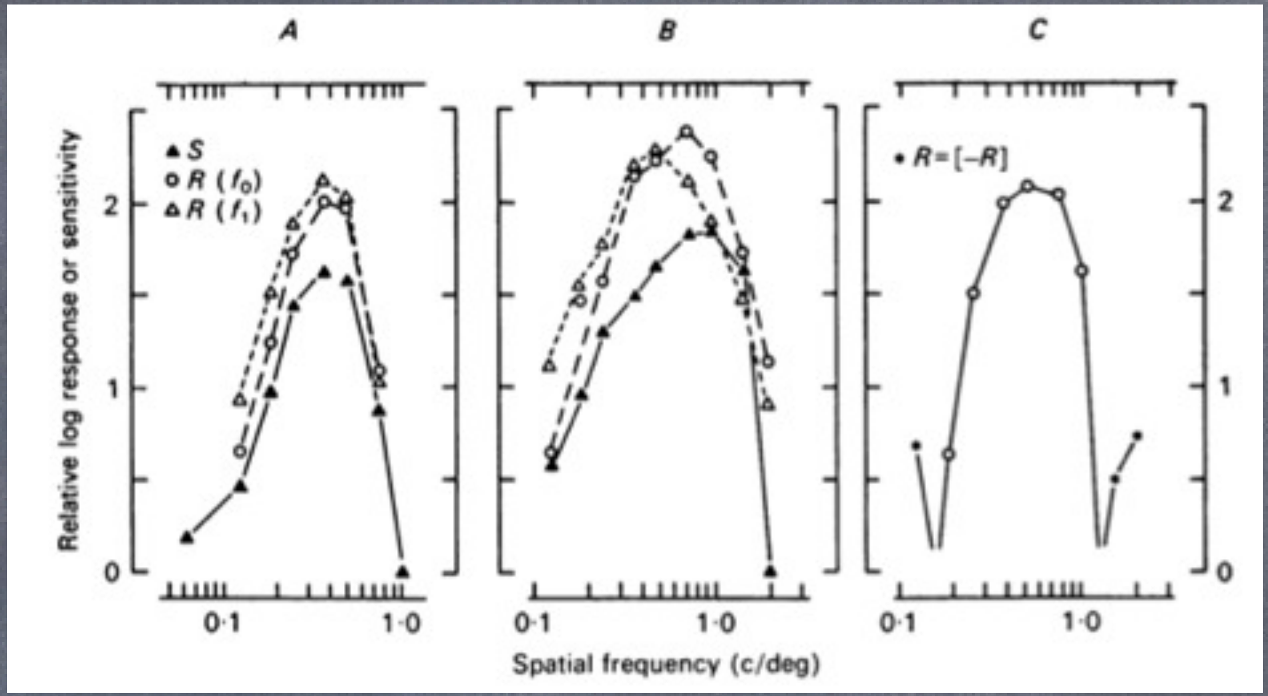
Organization of the spatial frequency preference

Perception of Spatial Frequency



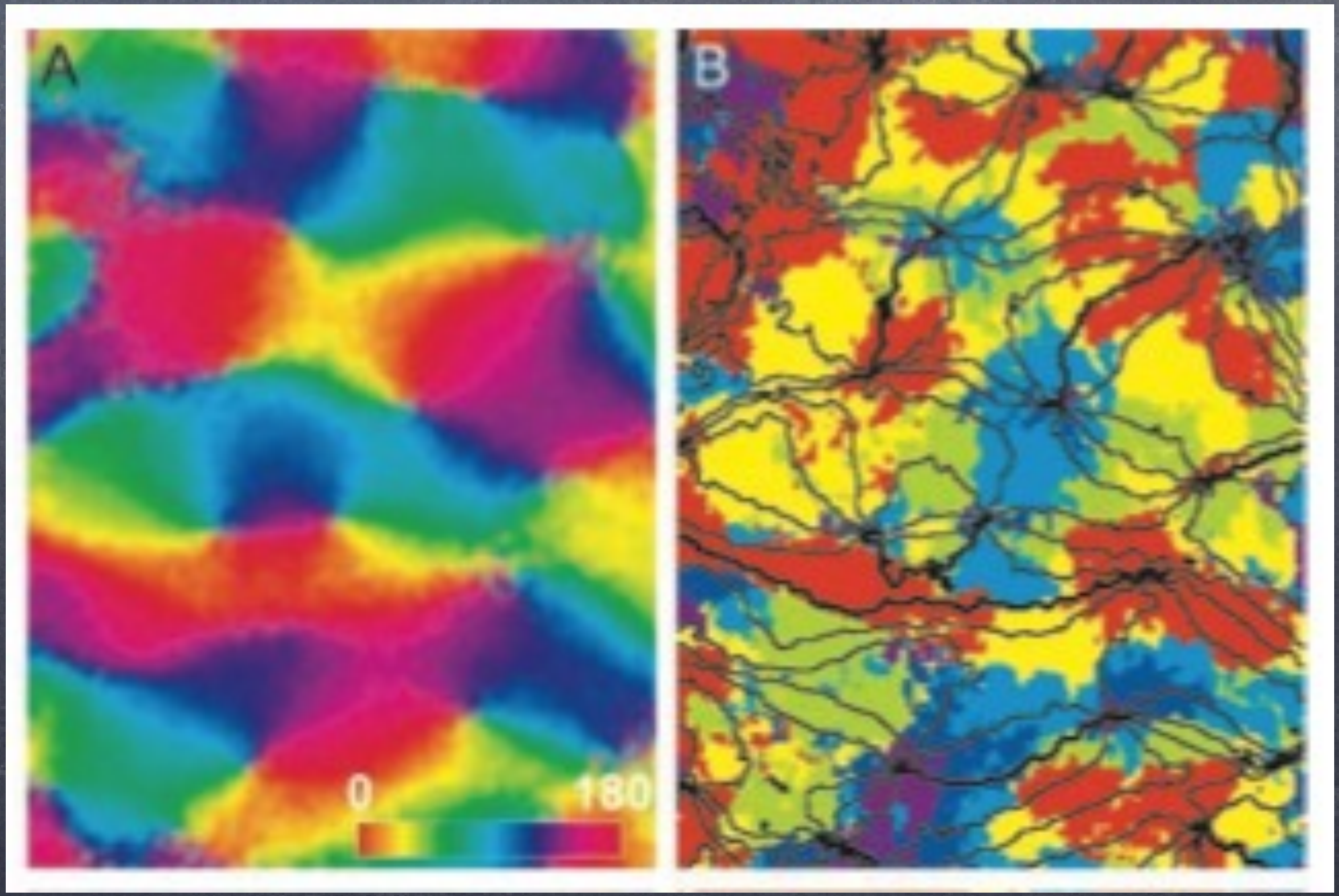
Previous results

Movshon et al,
J. Physiol (1978)



Cat

Issa et al,
J. Neurosci (2000)



Cat

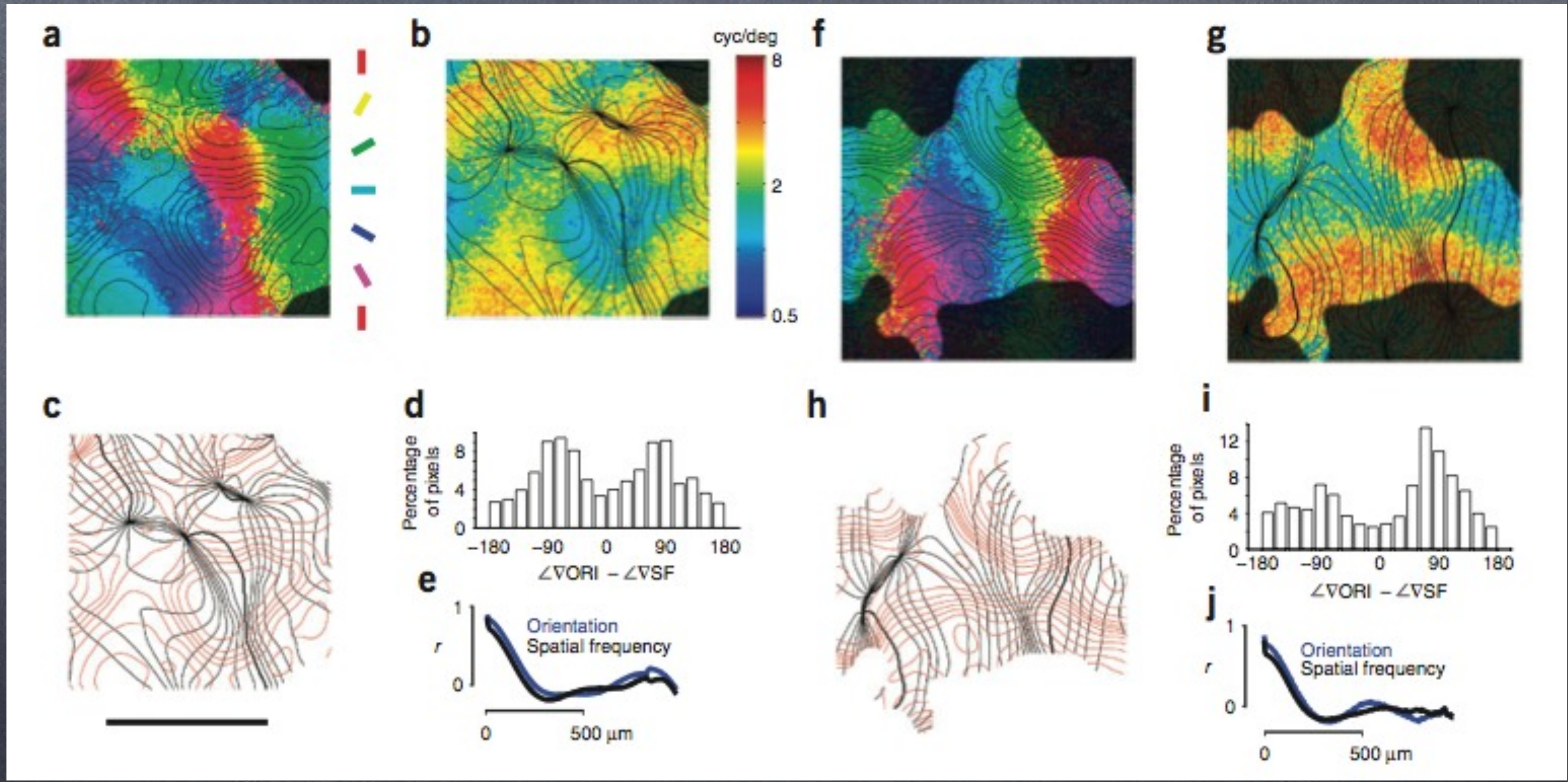
Uniform coverage
Continuity-coverage compromise

Obermayer et al, PNAS (1990); Durbin and Mitchison, Nature (1990)

Swindale et al, Nature Neuroscience (2000); Swindale, Cerebral Cortex (2000)

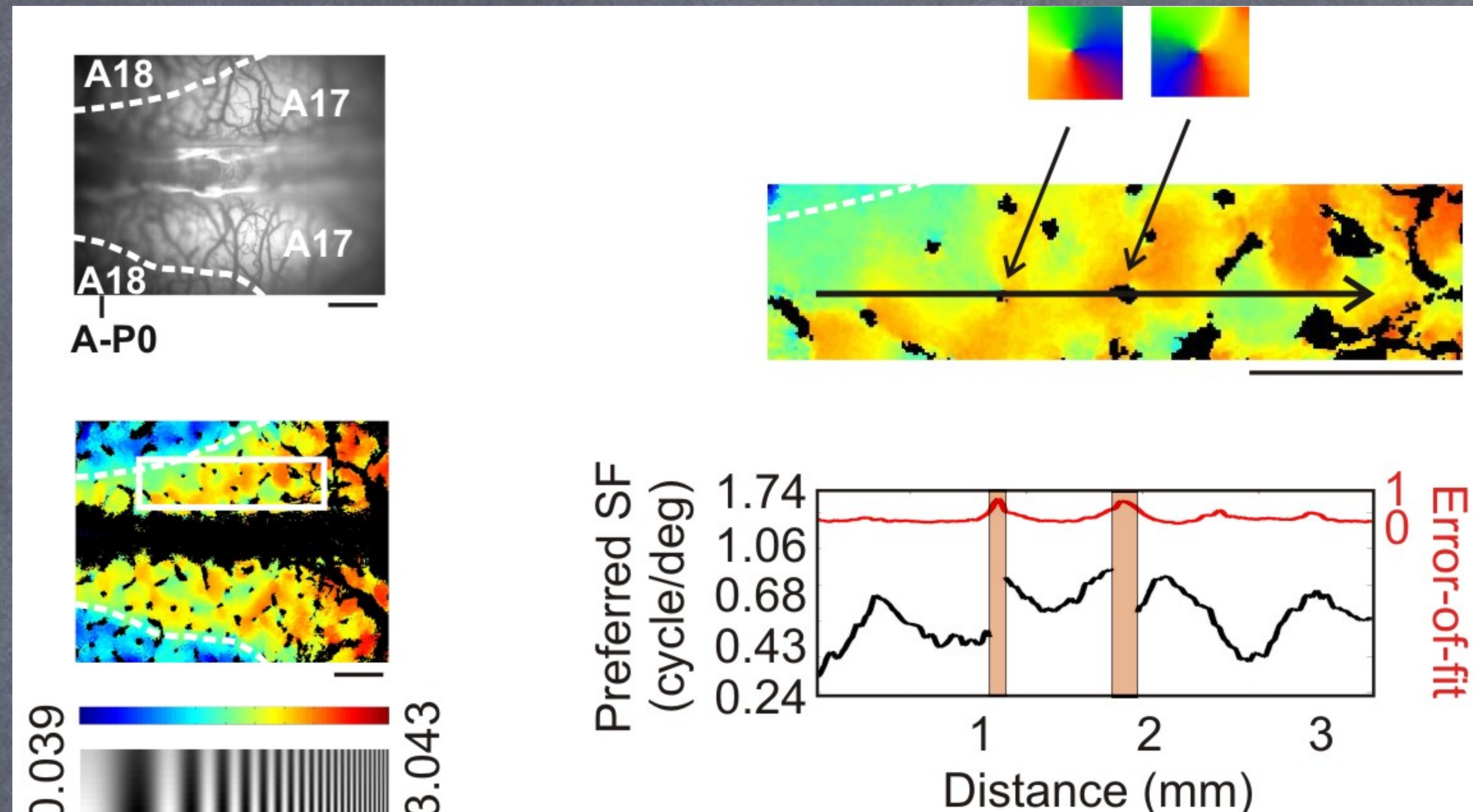
Nauhaus et al, Nature Neuroscience (2012)

Macaque

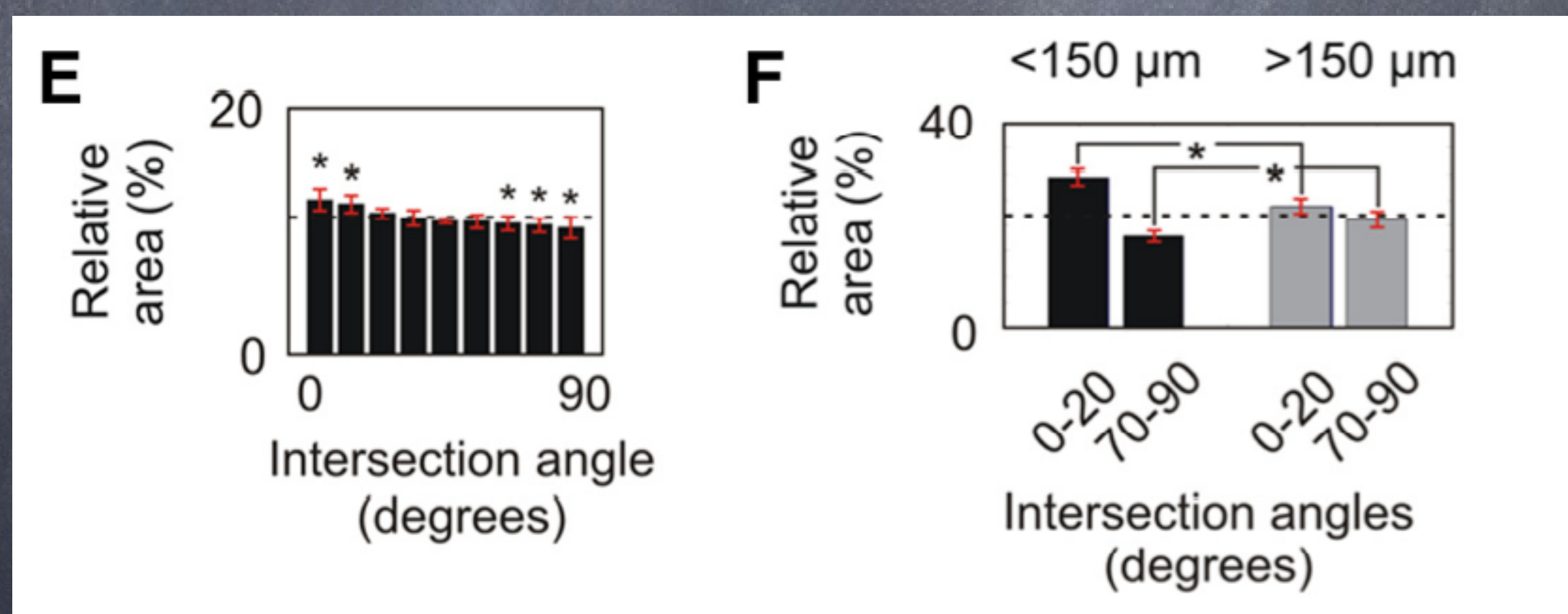


For cat, not clear orthogonal

Ribot et al,
J. Neurosci (2013)

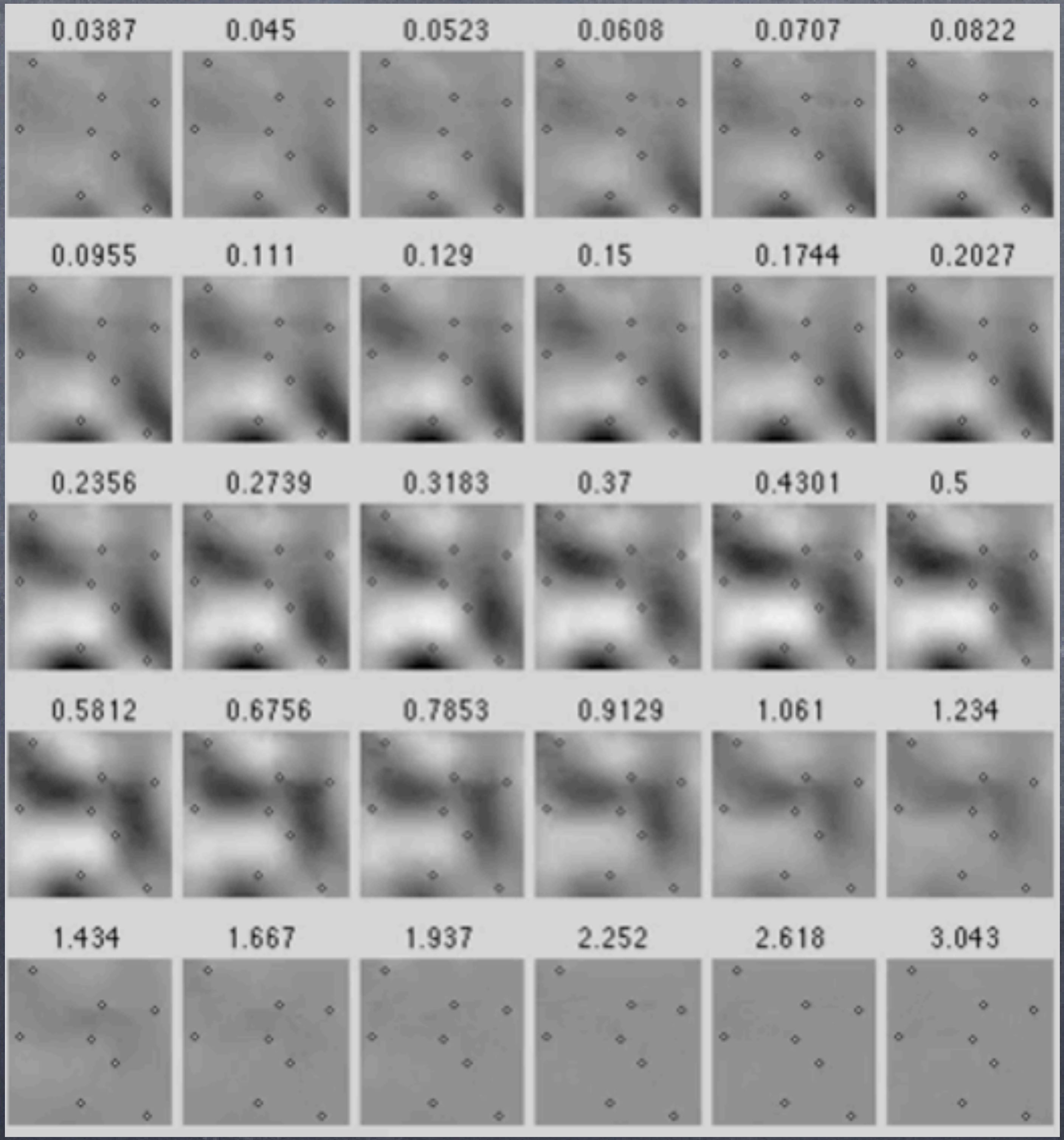


Cat

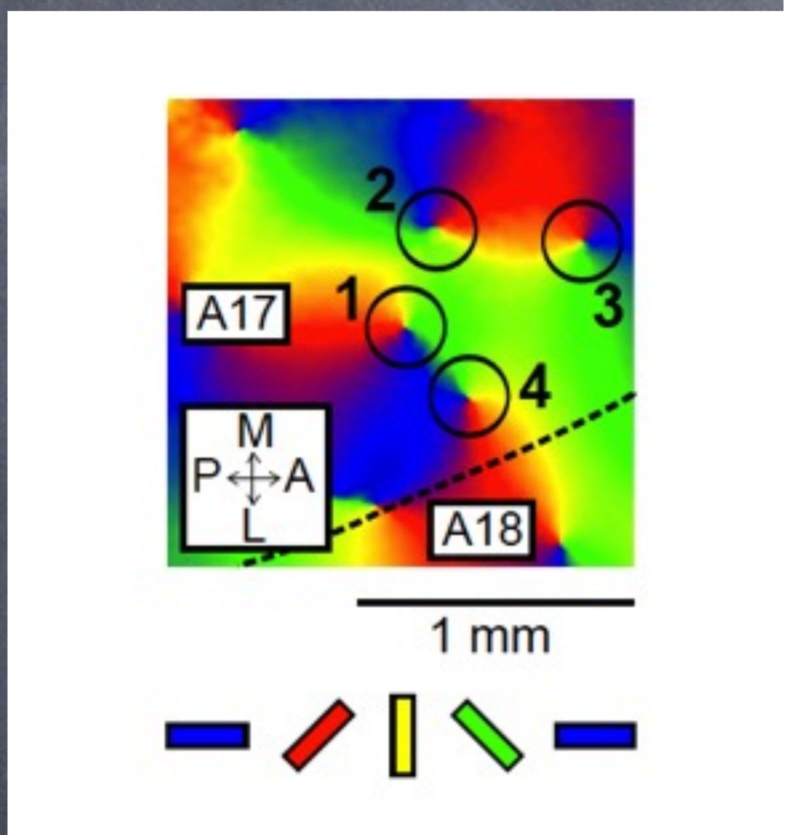


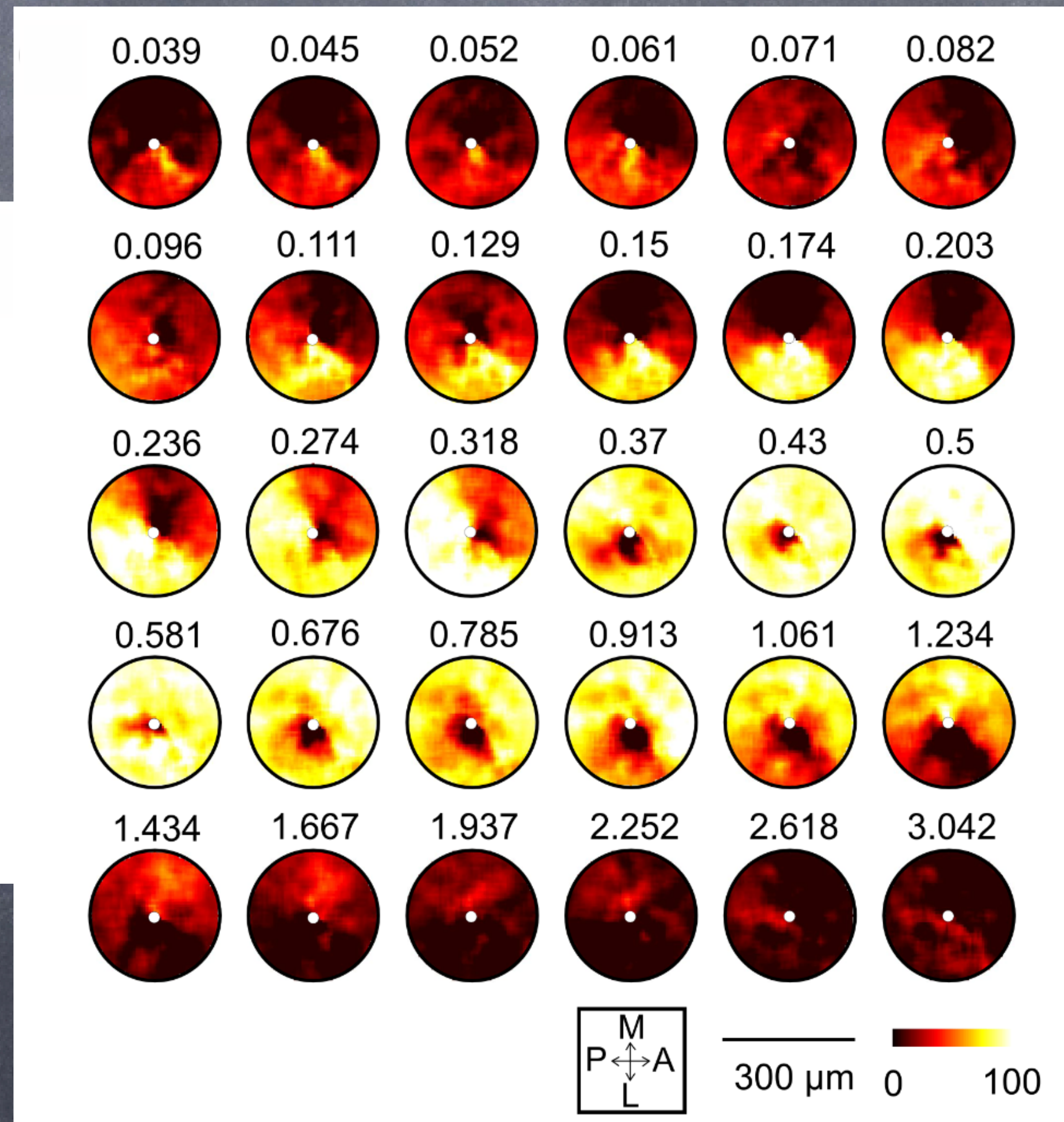
High-resolution optical imaging data

Raw data


0.0387
0.045
0.0523
0.0608
0.0707
0.0822
0.0955
0.111
0.129
0.15
0.1744
0.2027
0.2356
0.2739
0.3183
0.37
0.4301
0.5
0.5812
0.6756
0.7853
0.9129
1.061
1.234
1.434
1.667
1.937
2.252
2.618
3.043

OR & SF maps: same singularities





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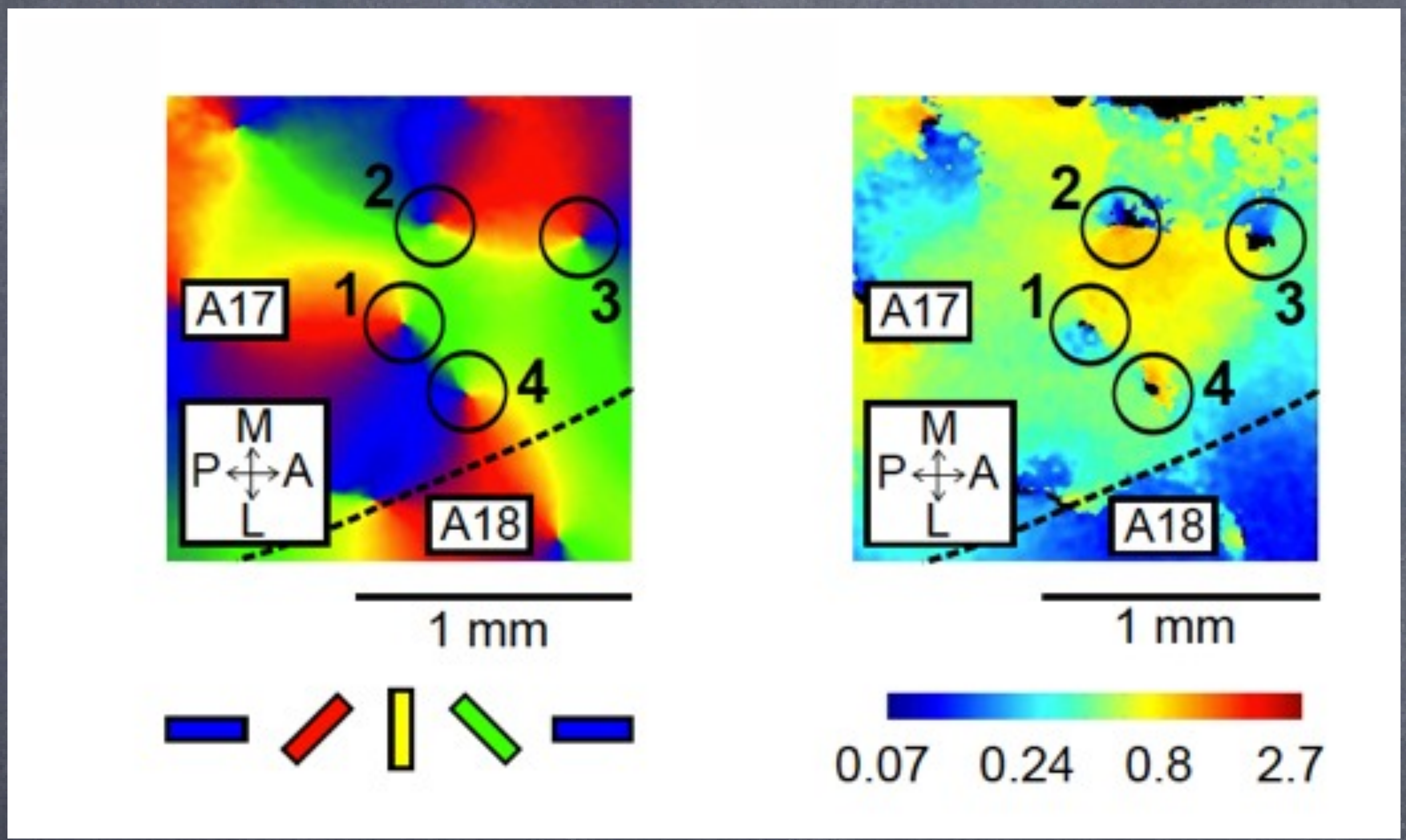

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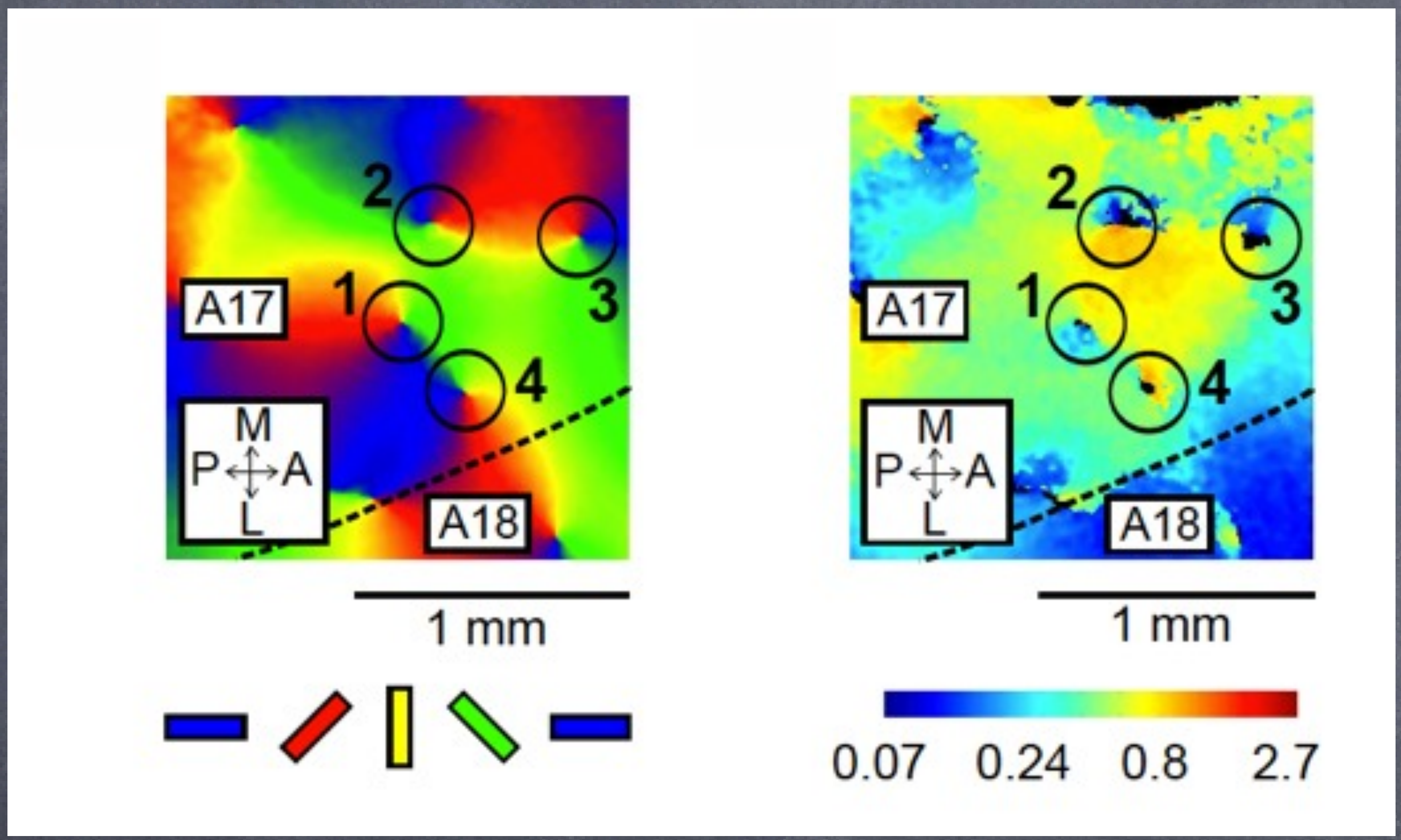
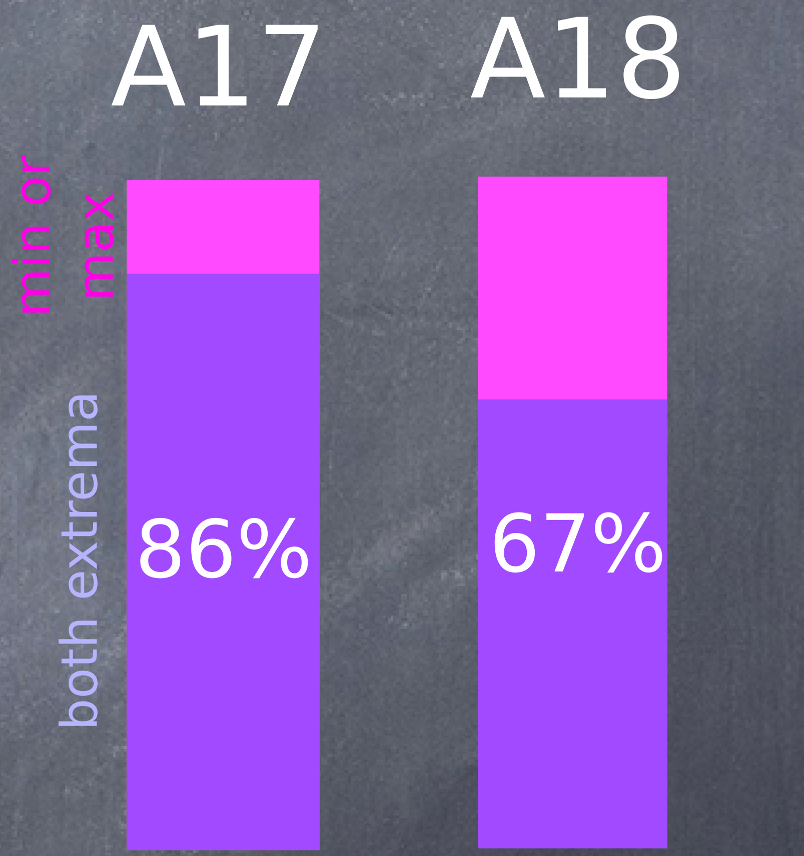
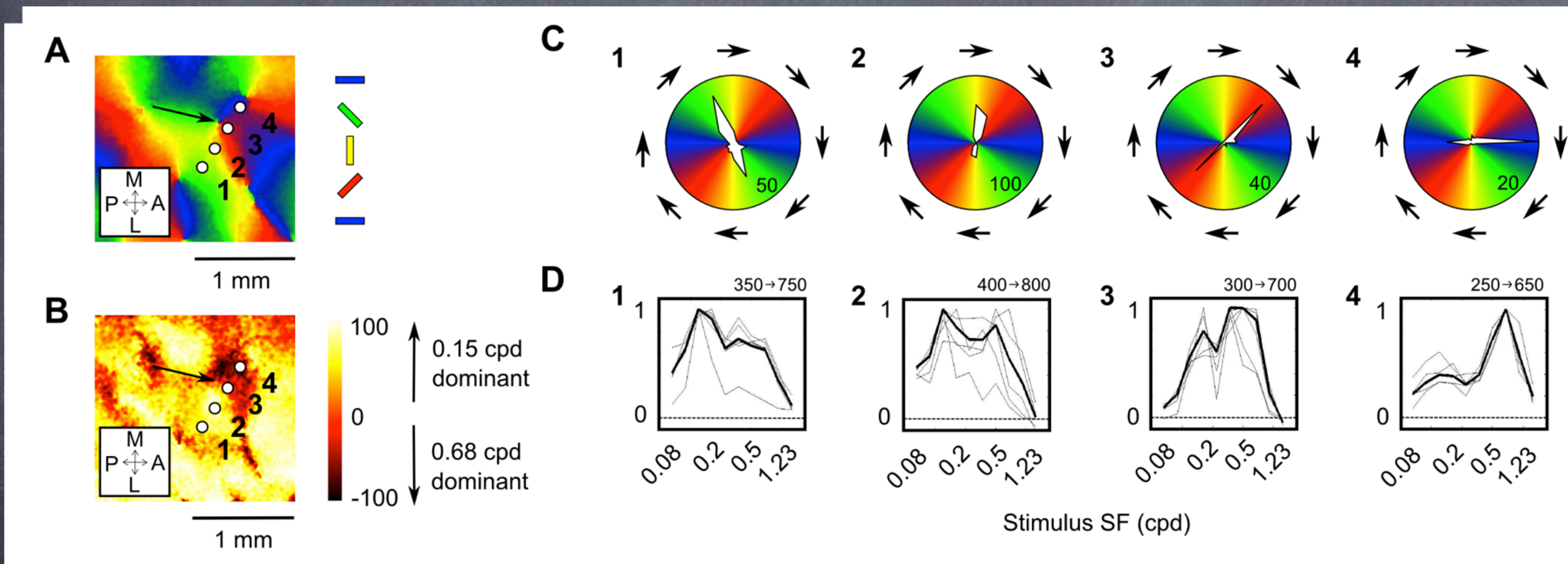


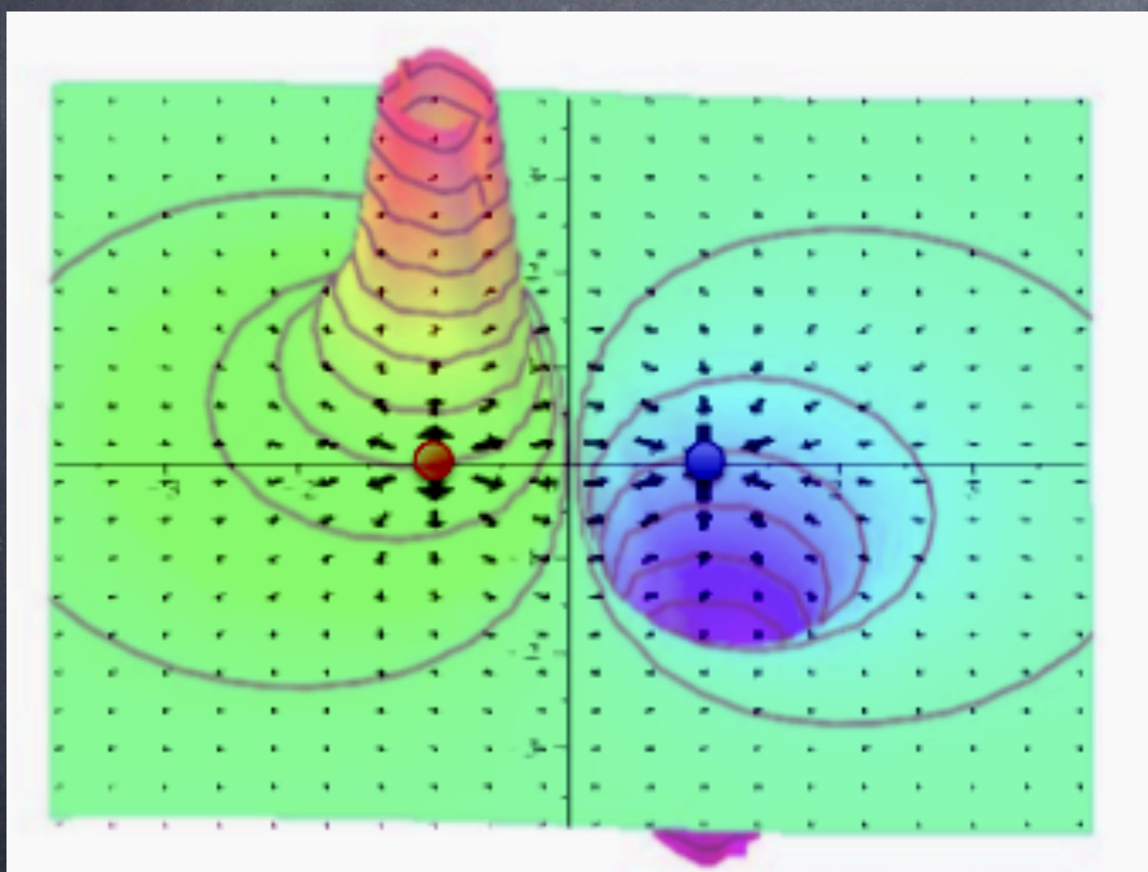


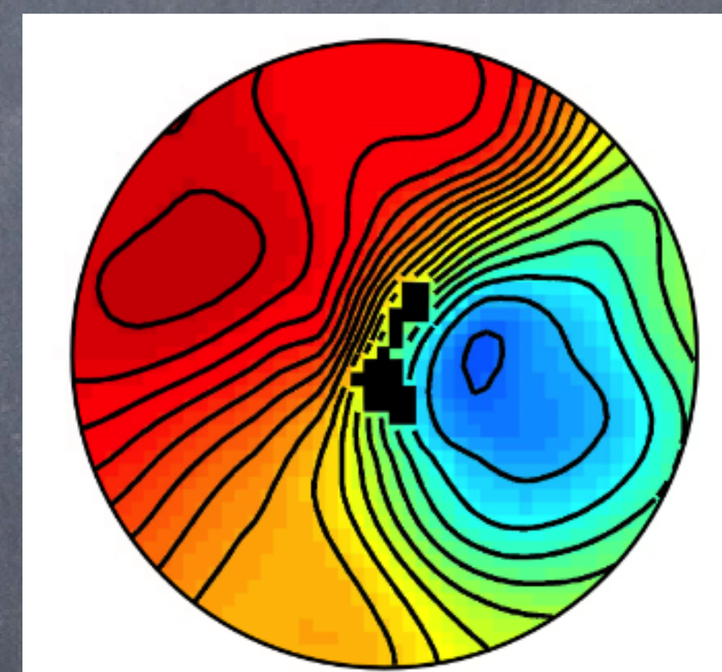
Statistics: 86% (59/69) of the PCs in A17 and 67% (51/76) in A18 present **BOTH** global **maximum** and global **minimum**.
The others present one global extremum.

Confirmation from electrophysiology



The analogy with electric dipole potential





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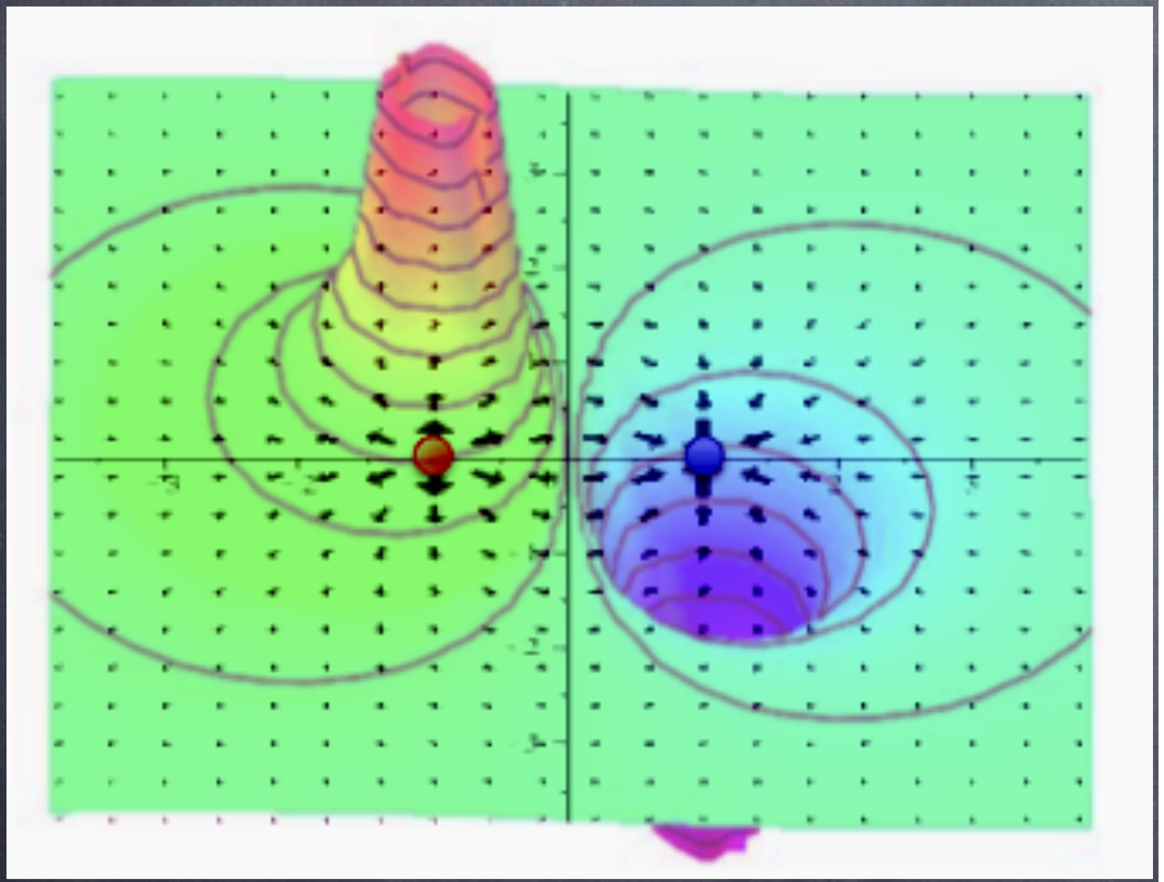
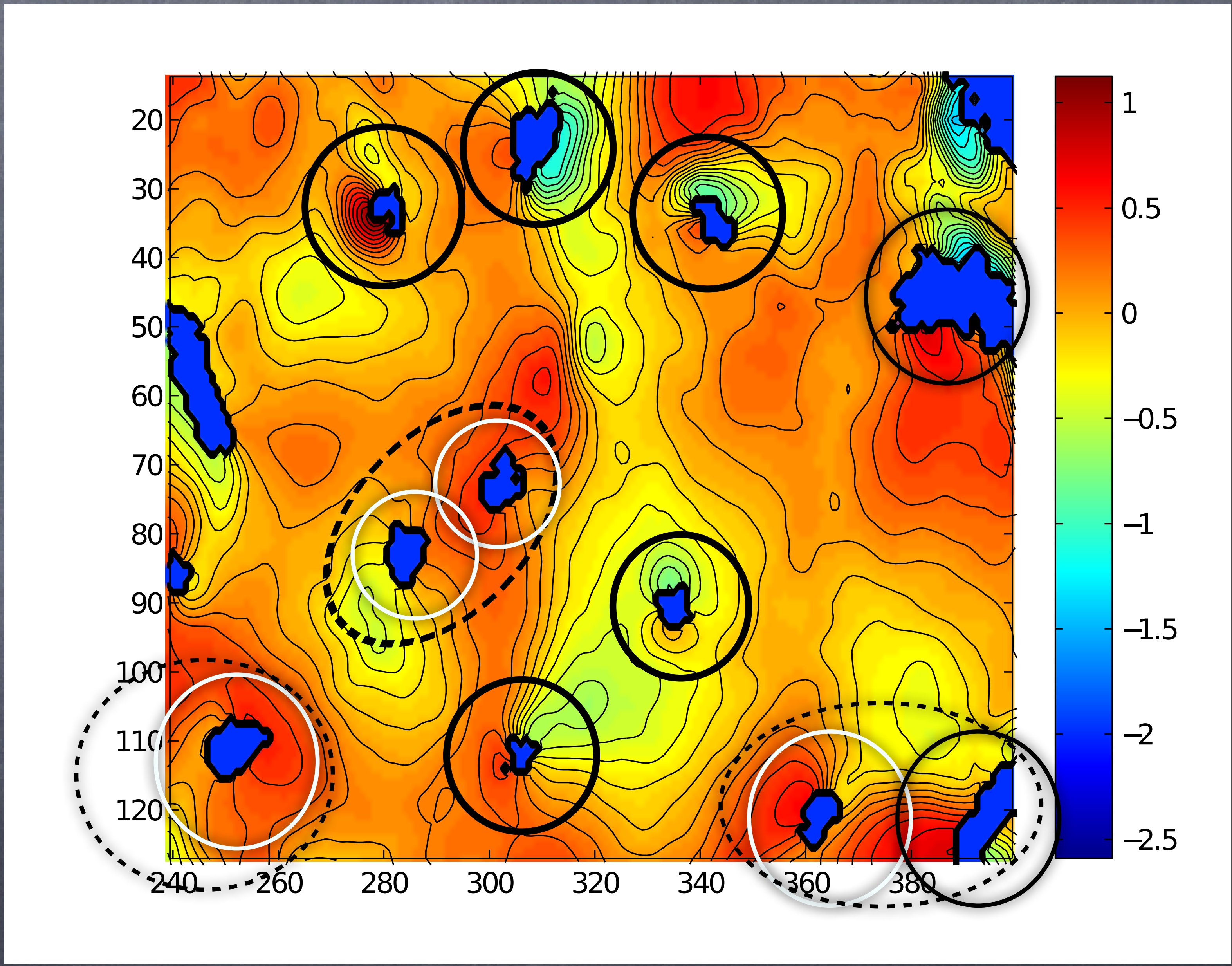
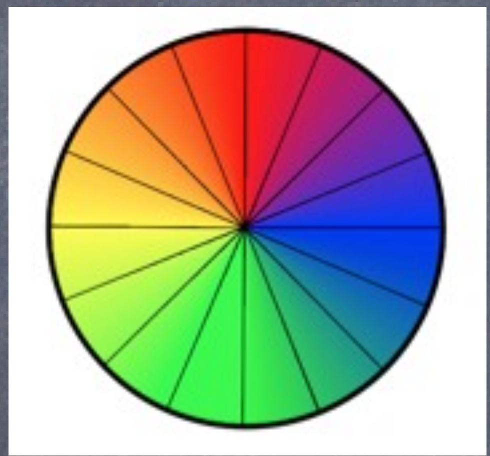




Universal Mathematical Properties of the Pinwheel-Dipole Topology

Organizing Principles

Properties of the Pinwheel Architecture

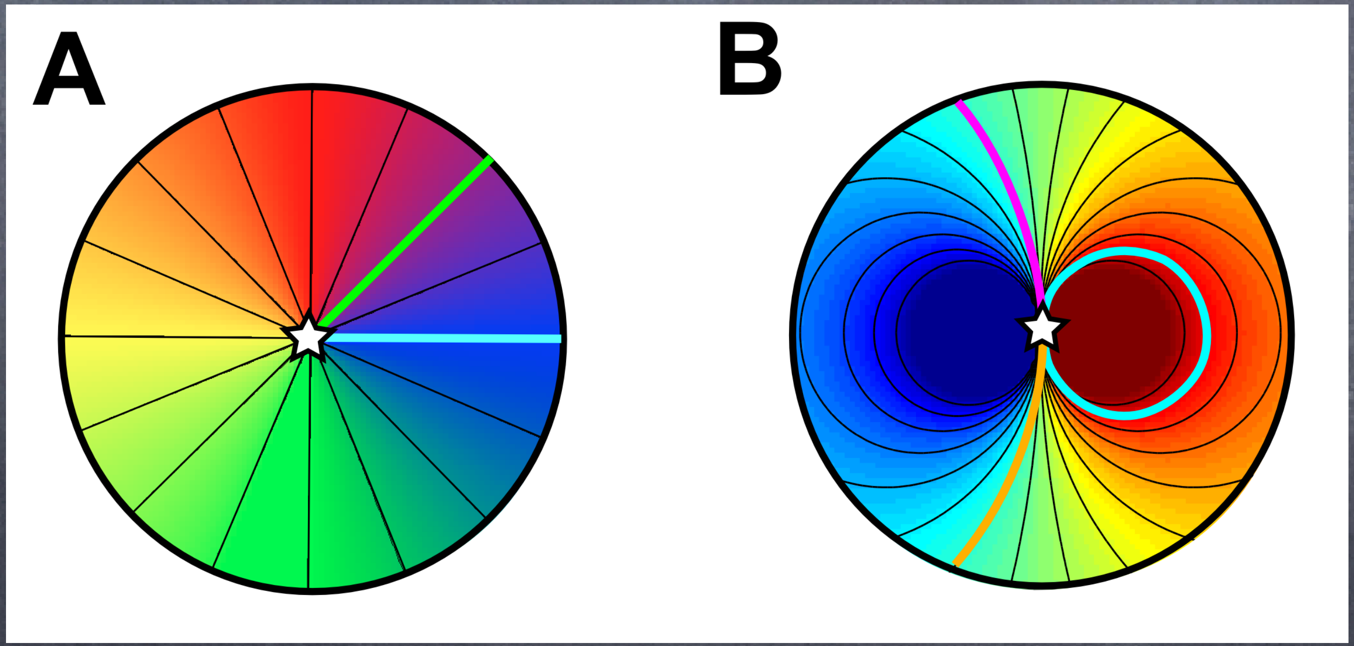


OR representation around PC
is **exhaustive** and **parsimonious**

Do these "organizing principles" extend to dipoles?

The geometric redundancy

Definition: The geometric redundancy is the maximal number of connected components of level sets of the map.



Geometric redundancy = 1 Geometric redundancy = 2

Pinwheel is the universal topology for angles

(under local exhaustivity and parsimony)

Proposition 1. *A continuous surjective map from the disc to the circle, with geometric redundancy 1, has the topology of the pinwheel.*

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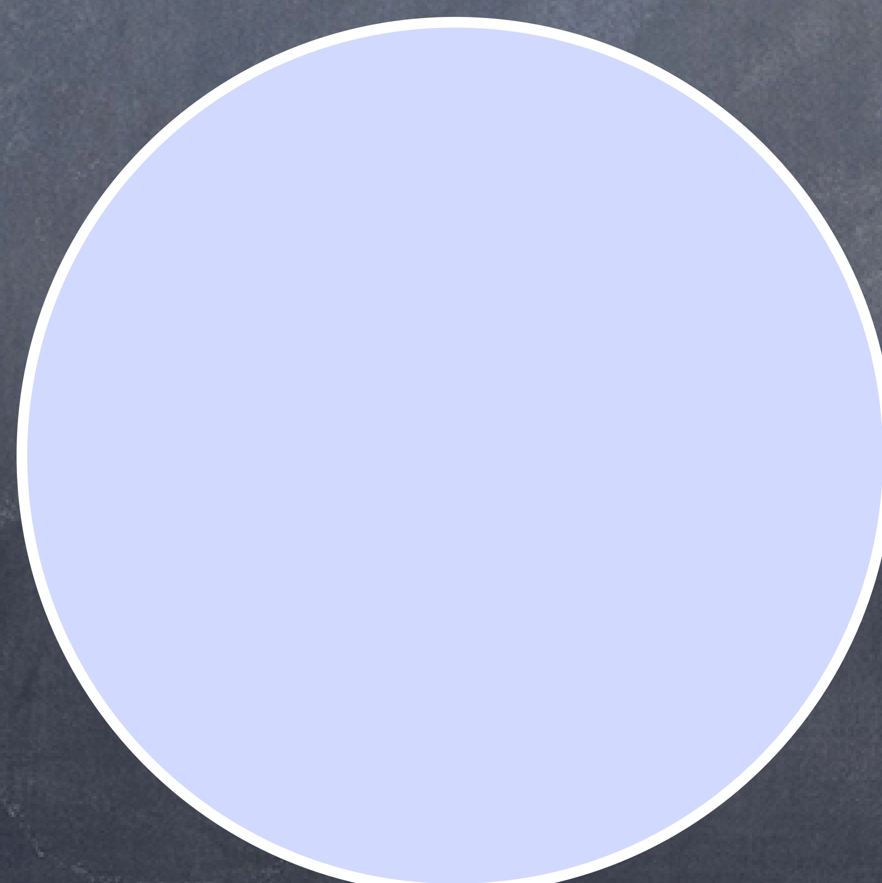

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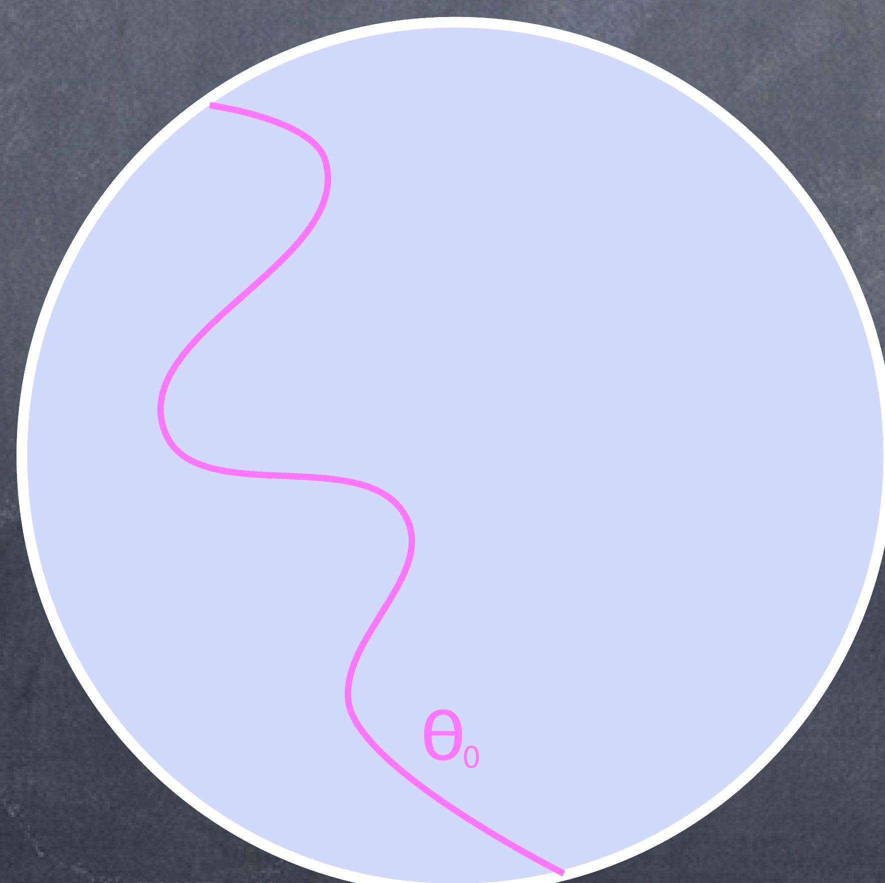

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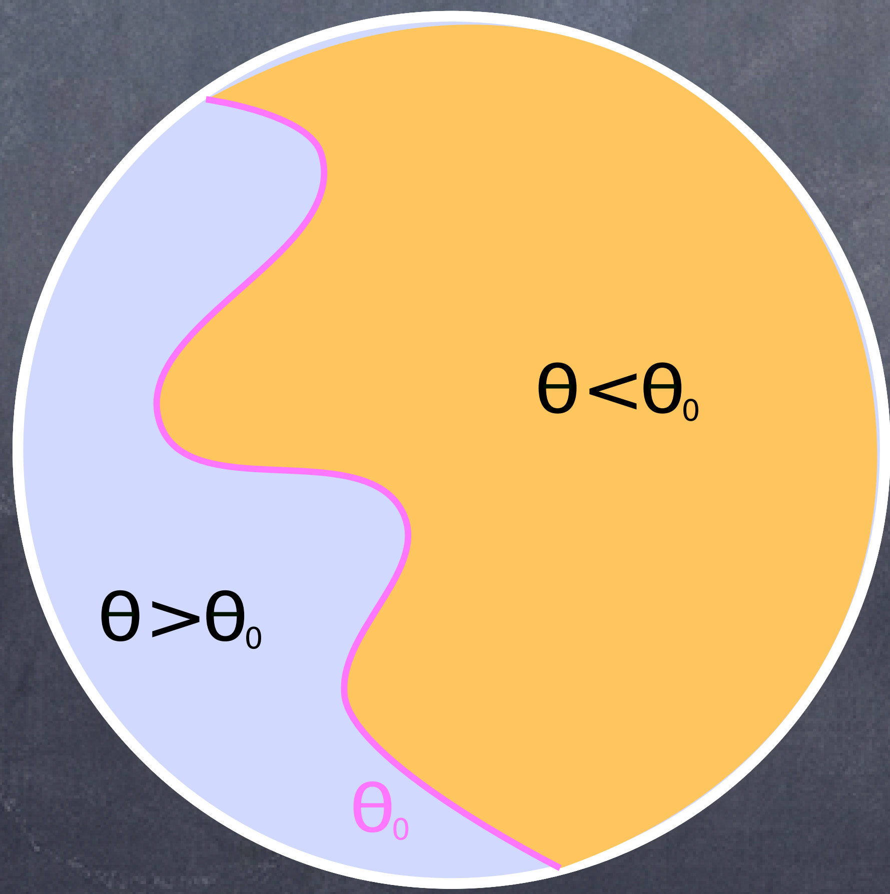

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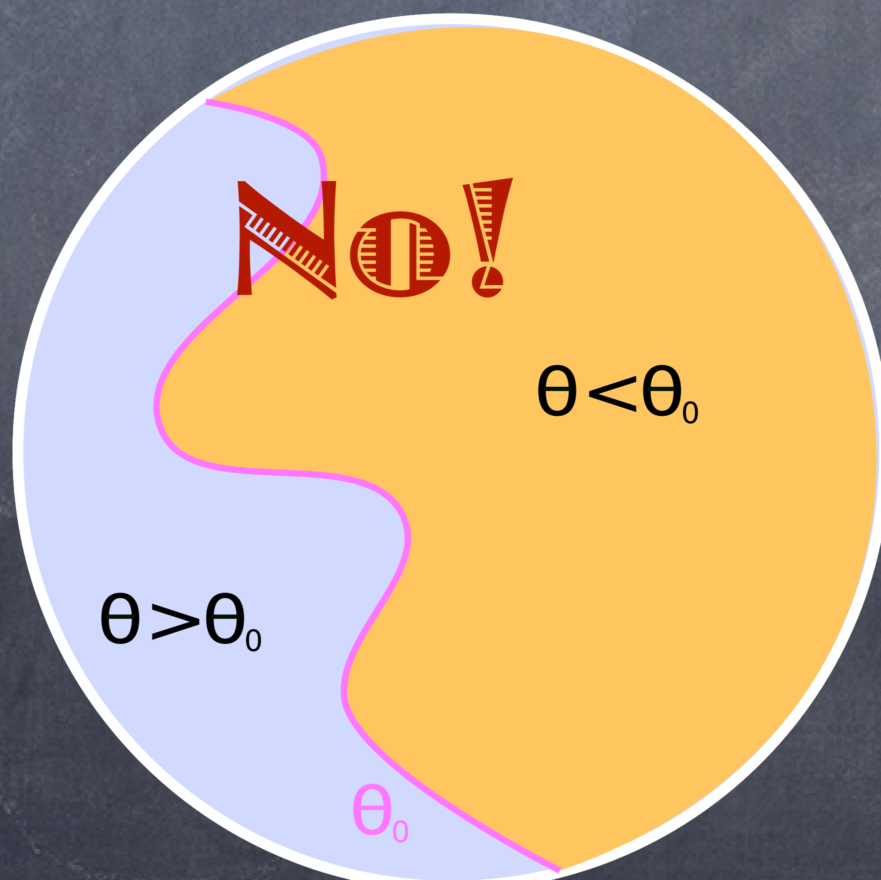

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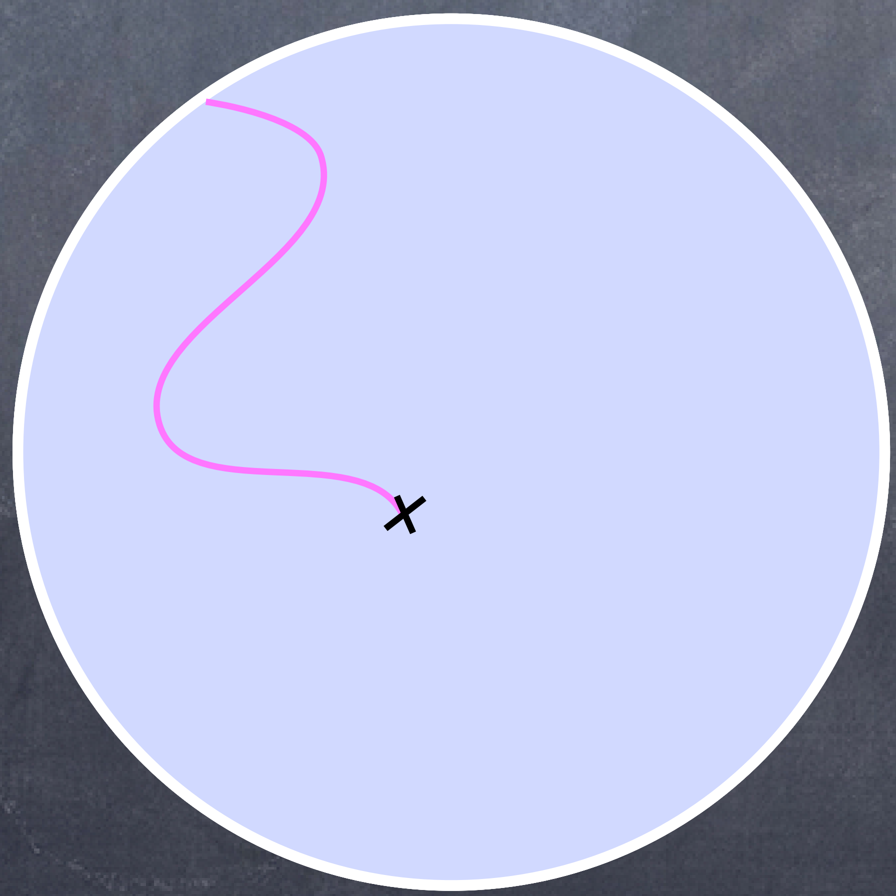

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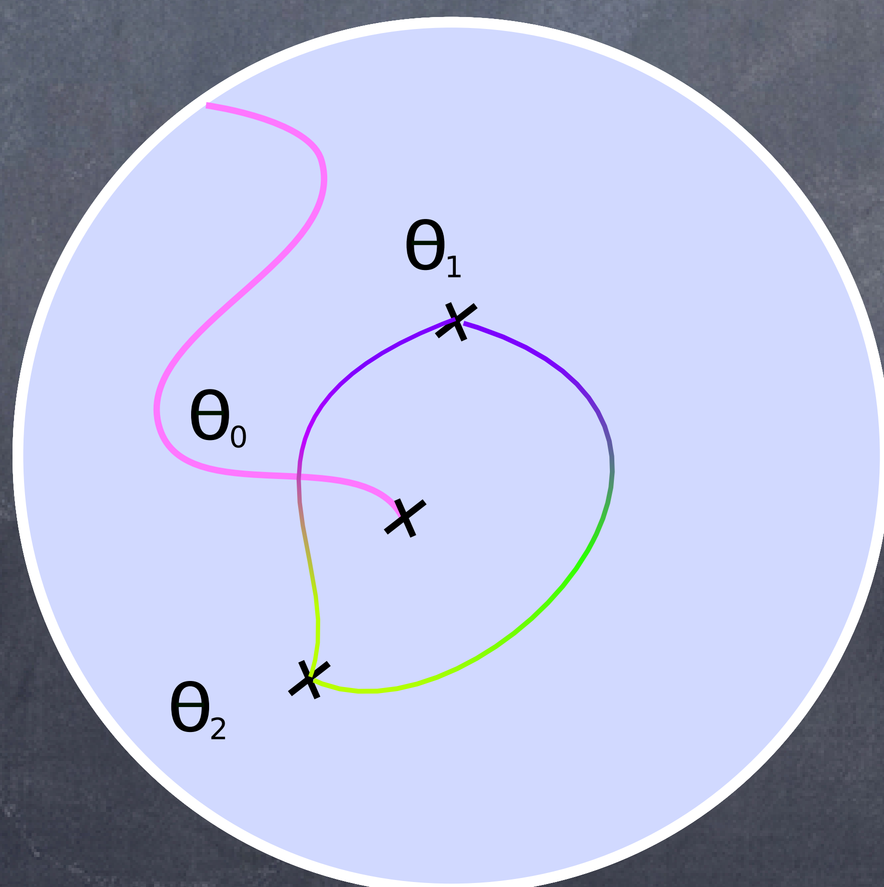

Pinwheel is the universal topology for angles

(under local exhaustivity and parsimony)

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Dipole is the universal topology for SF

For SF we consider an **open** interval, which is equivalent to considering continuous representations of the whole real line.

Continuous maps from the disc to $\mathbb{R}$ cannot be surjective without singularities. We consider the simplest maps with one singular point.

Regularity: the map is smooth.

Exhaustivity: the map is surjective in any neighbourhood of 0.

Parsimony: the topological redundancy is minimal at any scale.

Dipole is the universal topology for SF

(regularity, exhaustivity, parsimony)

Minimality

Proposition 1. Exhaustivity cannot exist without a singularity. On the pointed disc, exhaustive maps cannot have redundancy 1

Dipole is the universal topology for SF

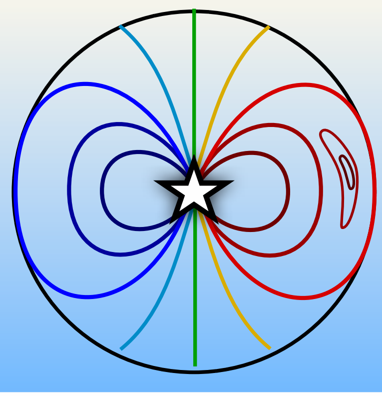
(regularity, exhaustivity, parsimony)

Minimality

Proposition 1. Exhaustivity cannot exist without a singularity. On the pointed disc, exhaustive maps cannot have redundancy 1

Uniqueness (apart from isolated defects)

Theorem. *A continuous map satisfying regularity, exhaustivity and parsimony, has the topology of the dipole.*



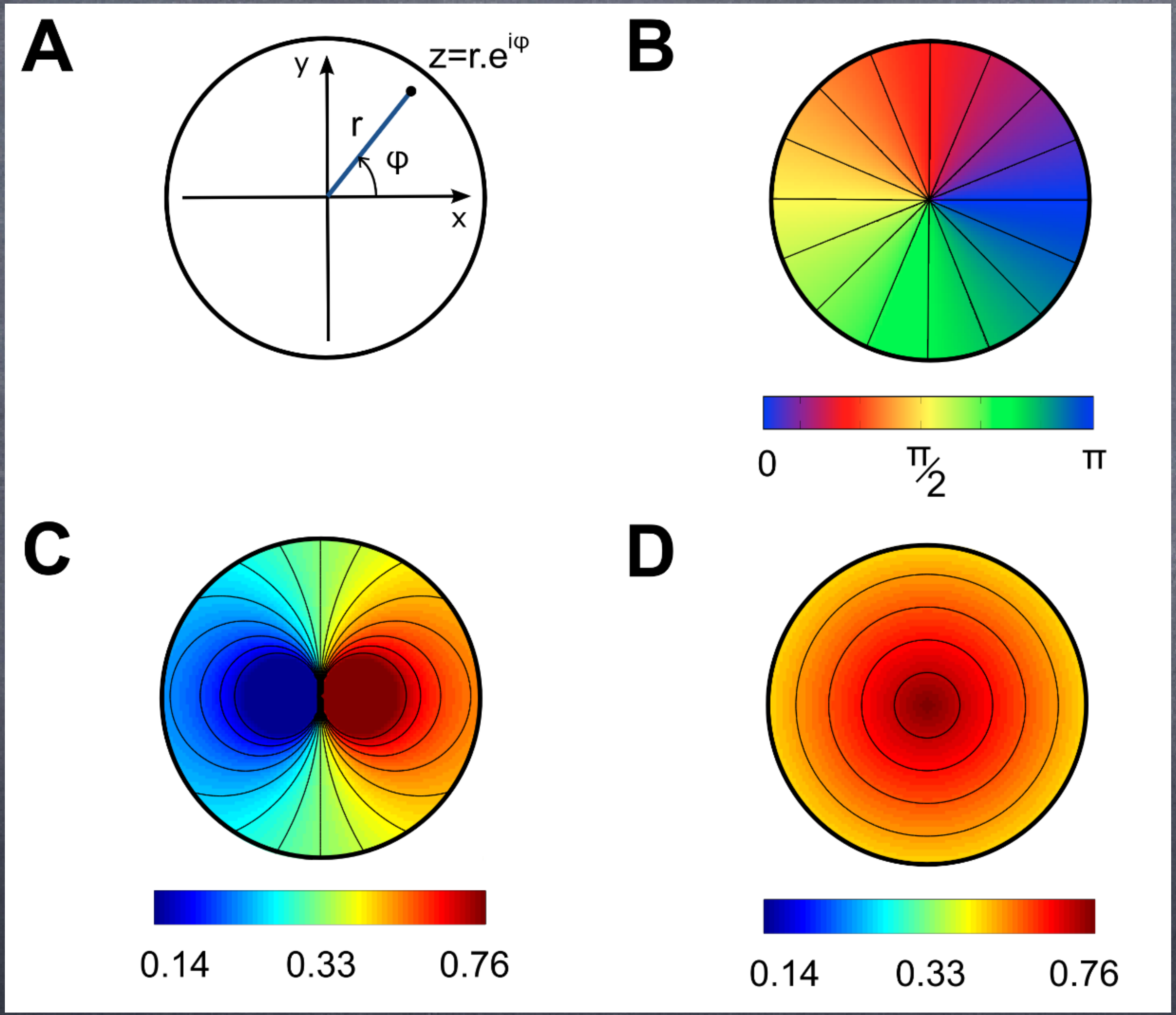
Idea of the proof: Exhaustivity implies the existence of an 8-shape bouquet, the rest can only be completed by 2 arcs connecting the singularity to the boundary

Dipole Models

Neuro-geometrical (local) model

OR pinwheels

$$OR = \frac{\varphi}{2} + \varphi_0$$

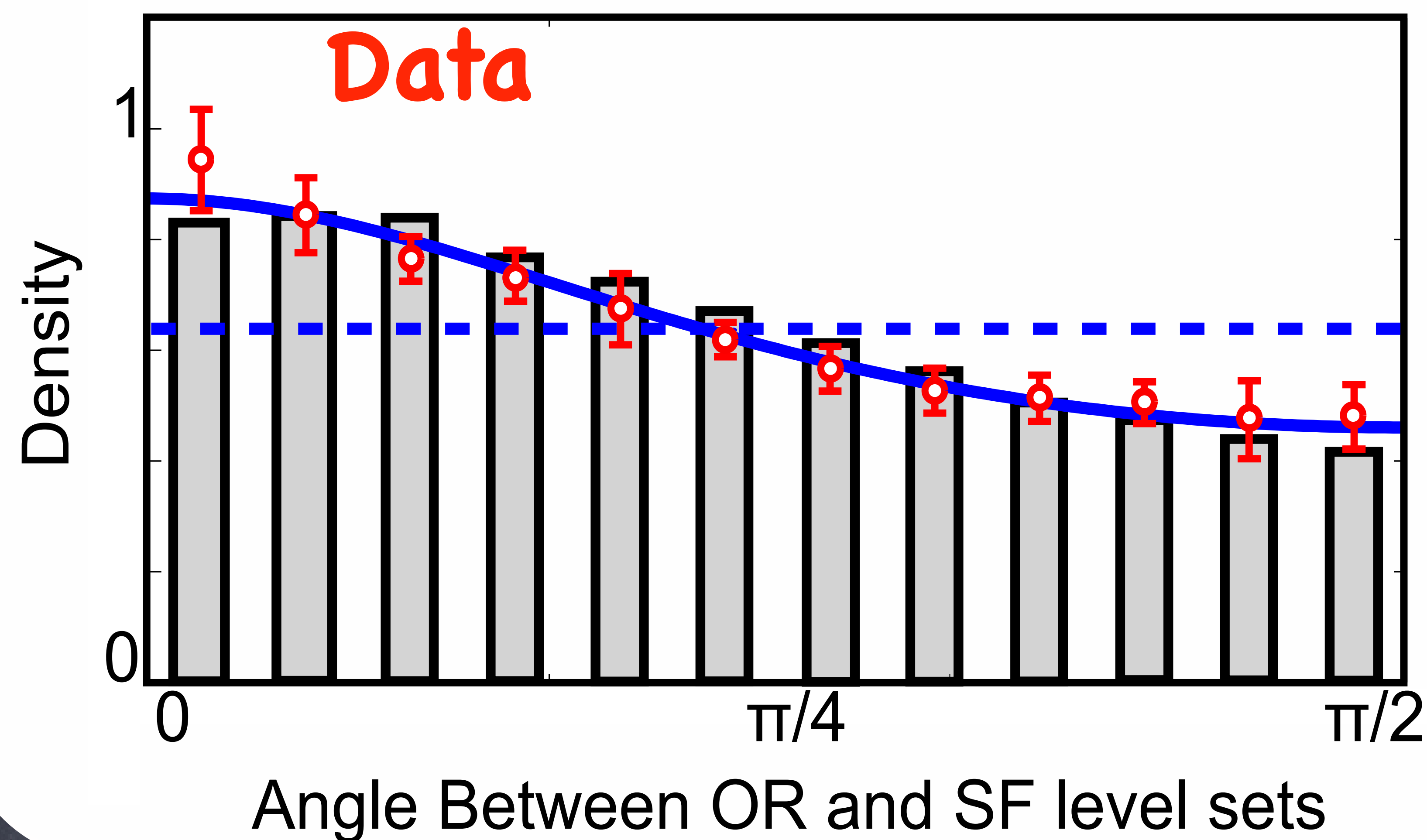


SF Dipoles

$$SF = \frac{\cos\varphi}{r^{\alpha}}$$

To compare: Orthogonal Architecture

Angles distribution fits data



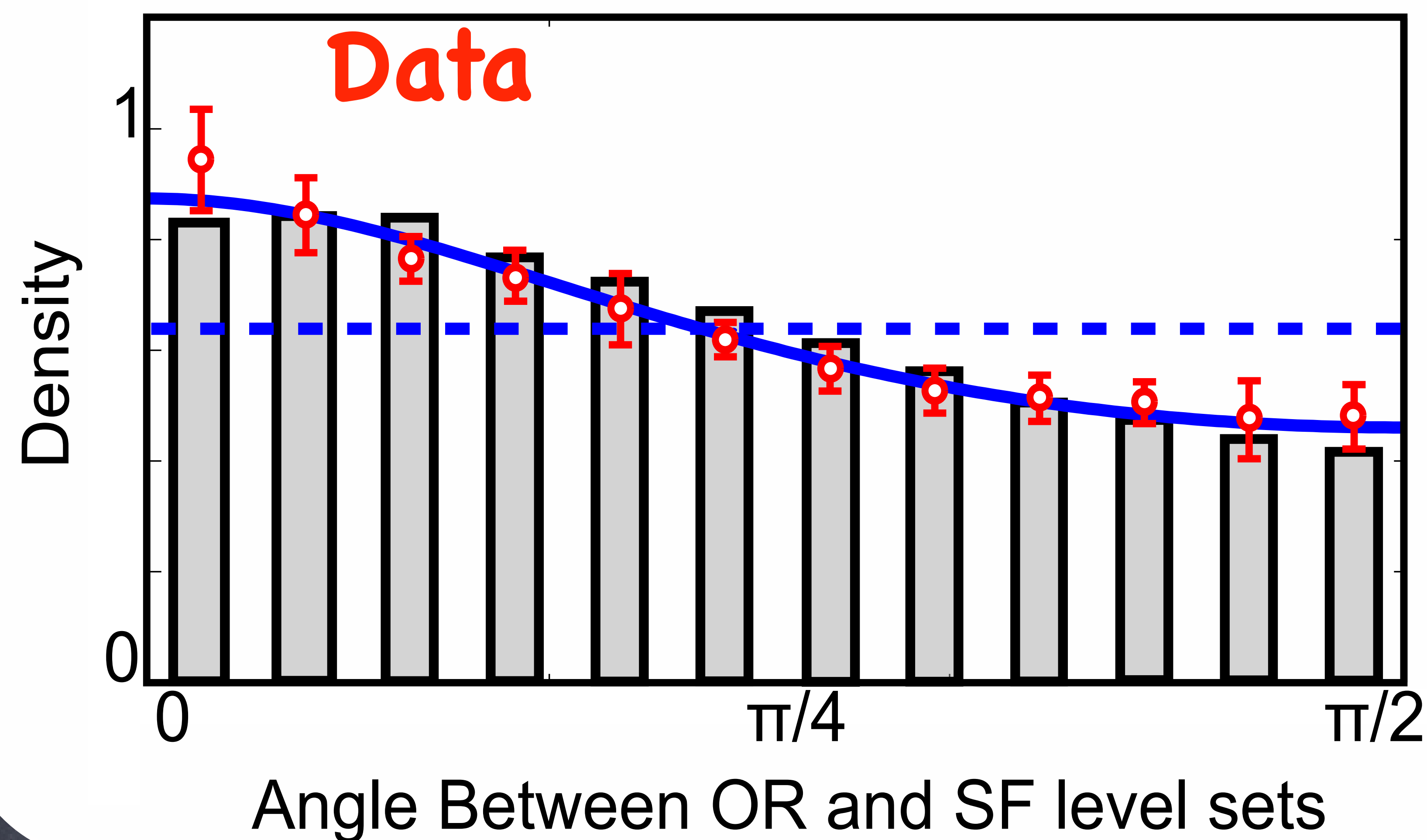
$SF : re^{i\phi} \mapsto \cos(\phi)/r^{\alpha}$

$\alpha = 1$

$\alpha = 0.73$

$$\cos\psi = \left| \frac{\sin(\phi)}{\sqrt{\alpha^2 \cos^2(\phi) + \sin^2(\phi)}} \right| \quad \Longrightarrow \quad P(\psi) = \frac{2\alpha}{\pi\left((\alpha^2 - 1)\cos^2(\psi) + 1\right)}$$

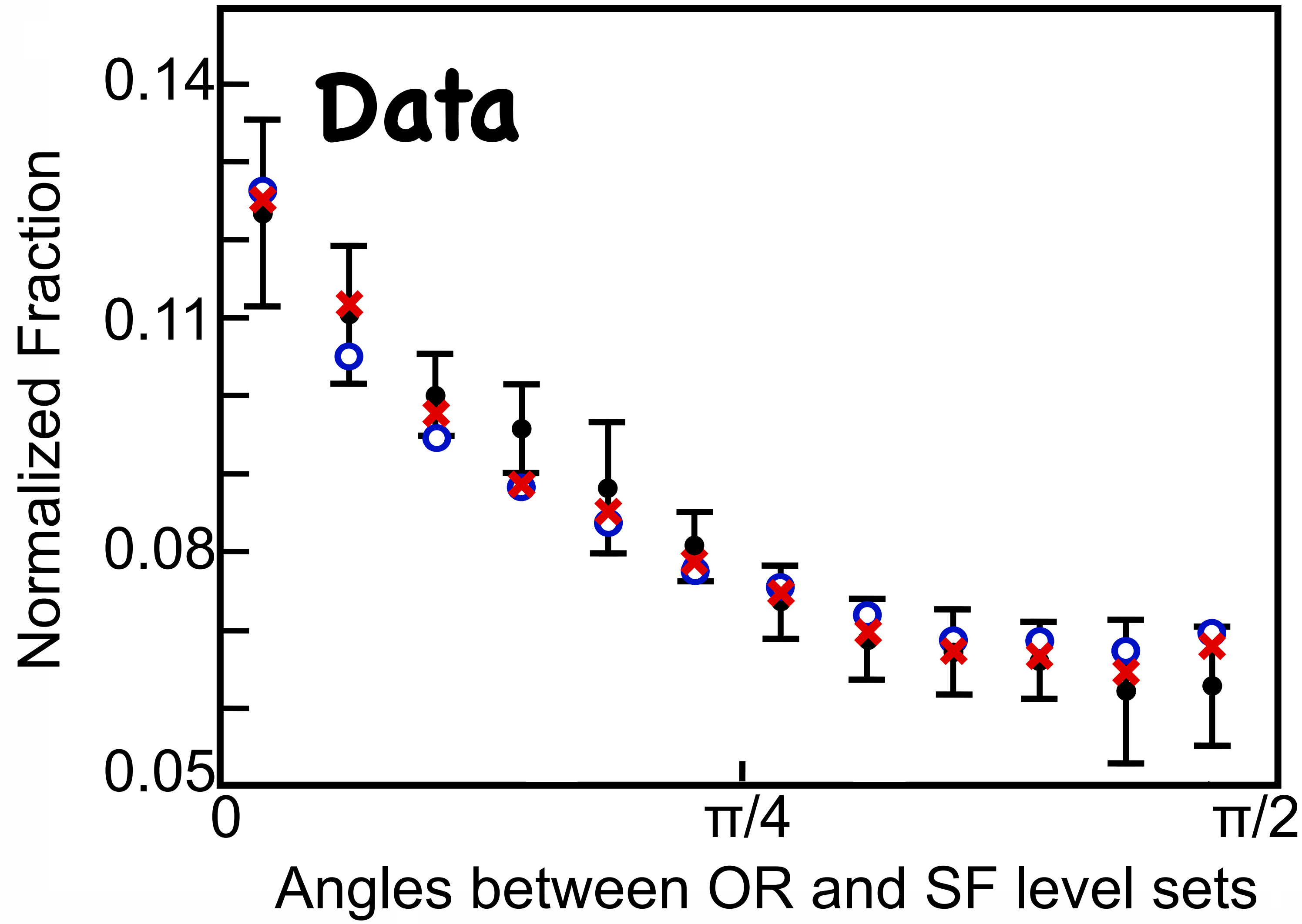
Angles distribution fits data



$A = 1.4$

$$re^{i\phi} \mapsto \max\left(-A, \min\left(A, \cos(\phi)\frac{R}{r}\right)\right)$$

Angles distribution fits data



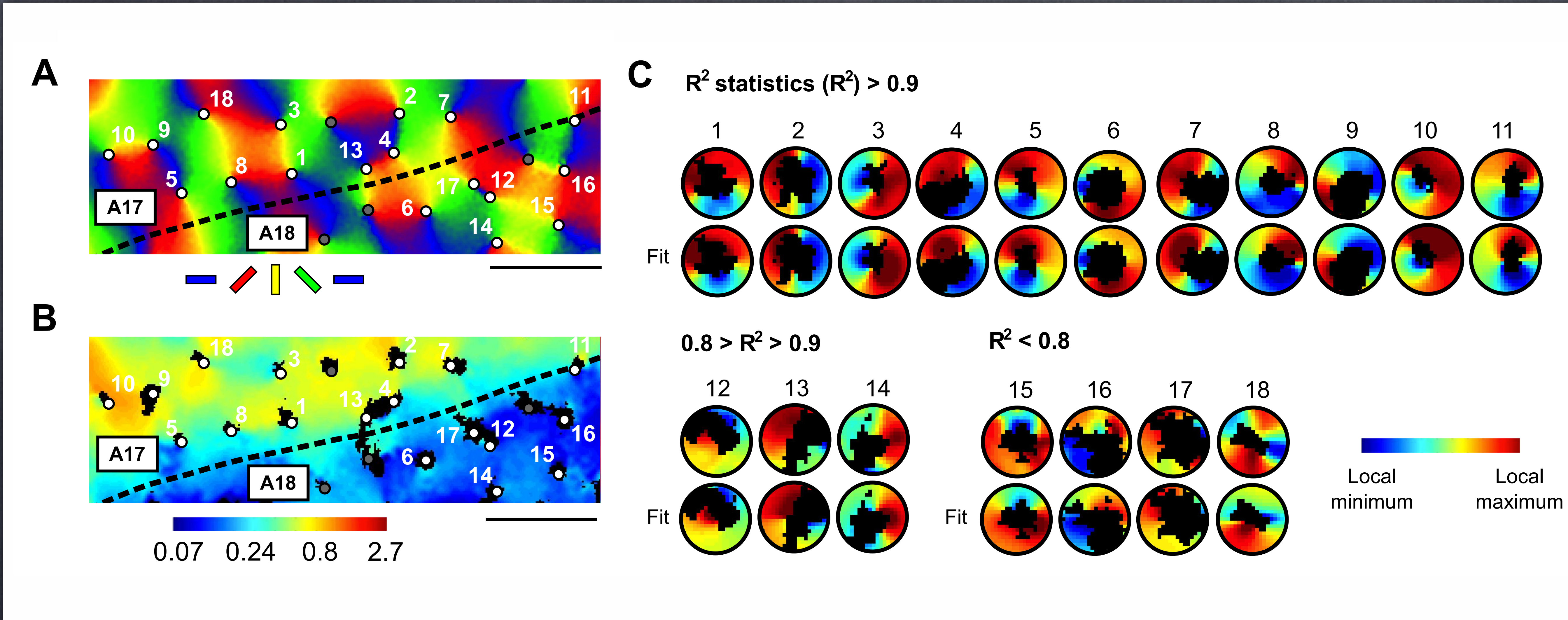
$B = 0.35$

$C = 0.3$

$$re^{i\phi} \mapsto \cos(\phi)\tanh\left(\frac{B\,R}{r}\right)$$

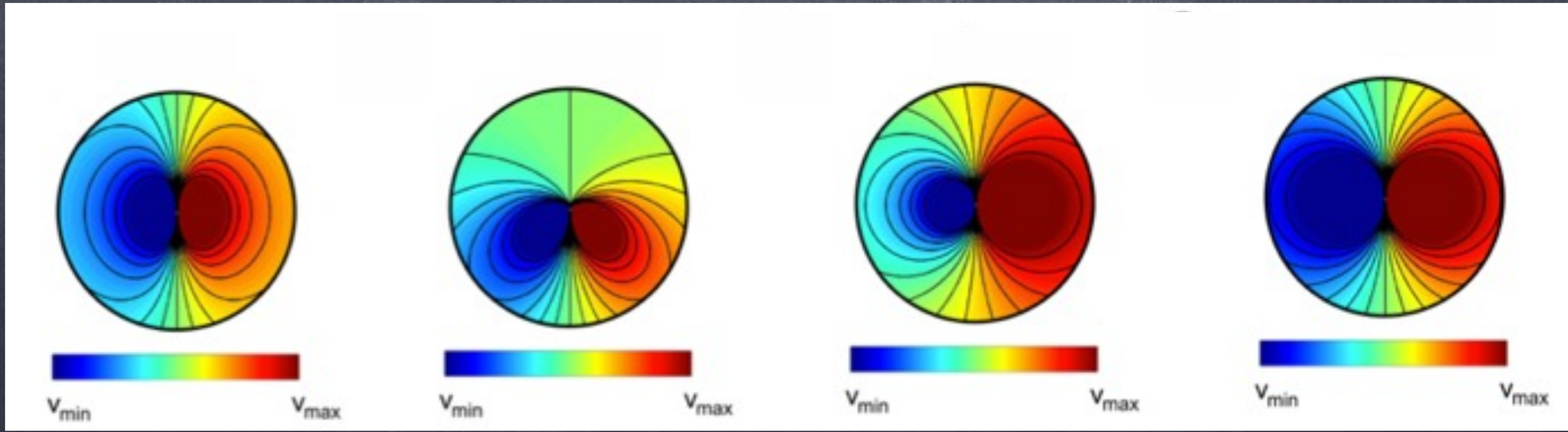
$$re^{i\phi} \mapsto \cos(\phi)\ \mathrm{erf}\left(\frac{C\,R}{r}\right)$$

Fit of the dipole model



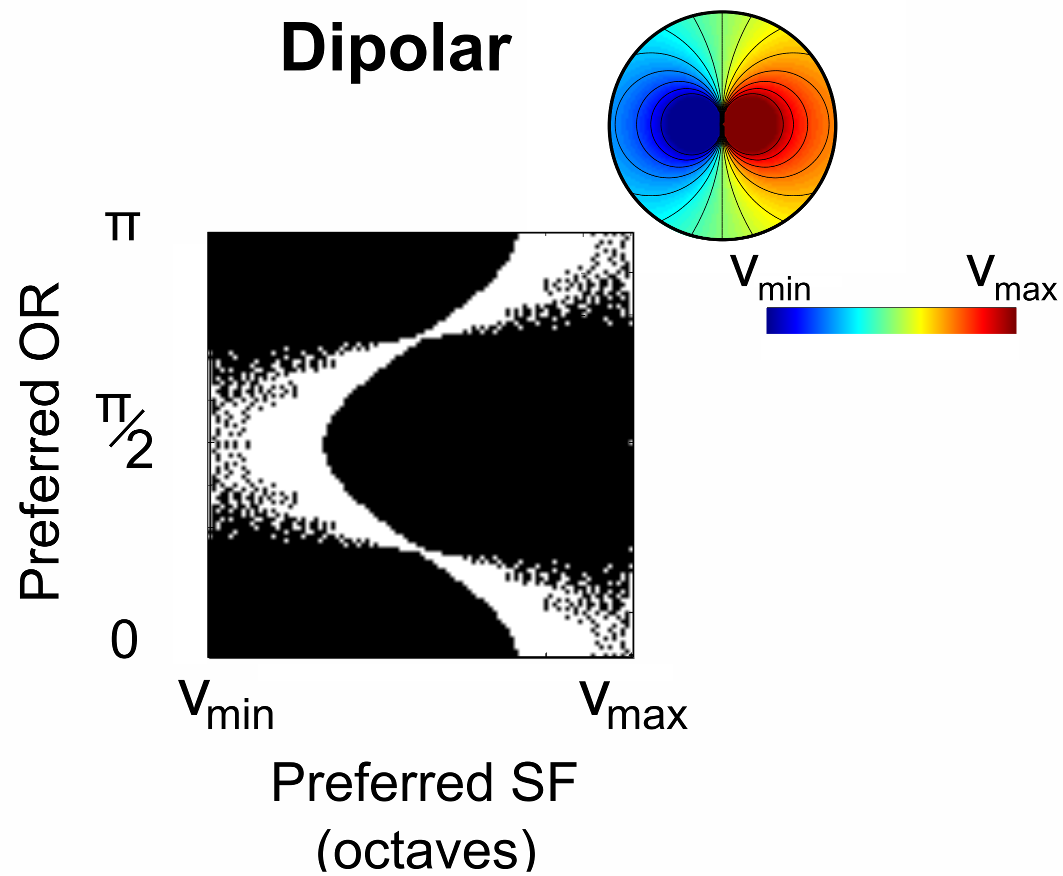
Taking into account angular variability

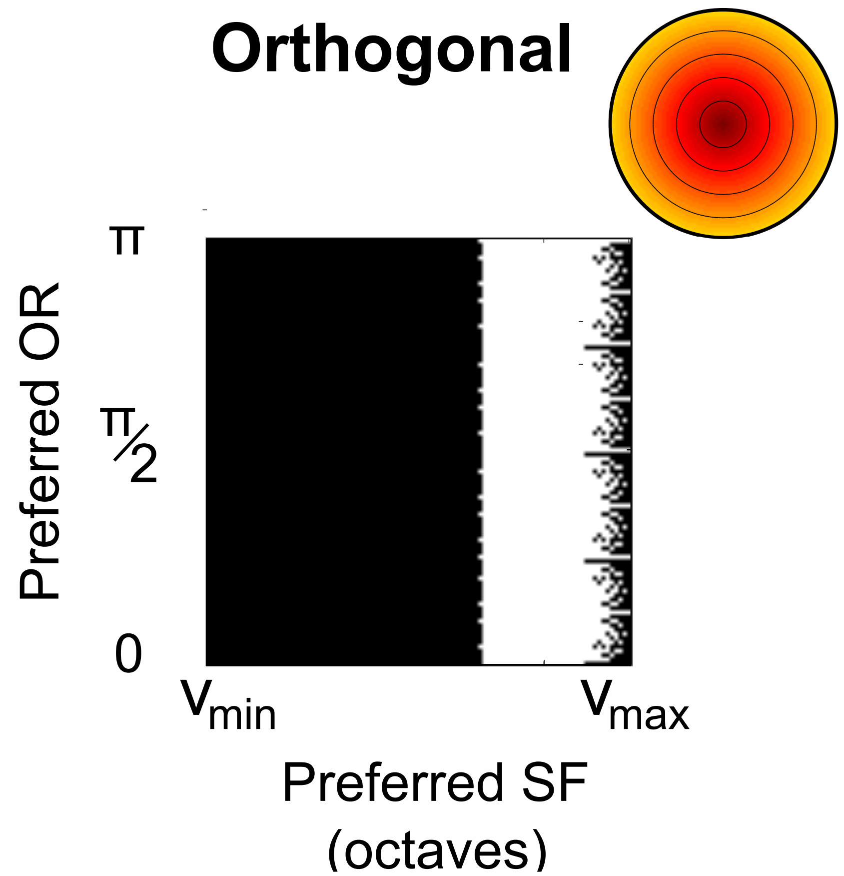
$$SF \sim \frac{\cos\varphi}{r}$$



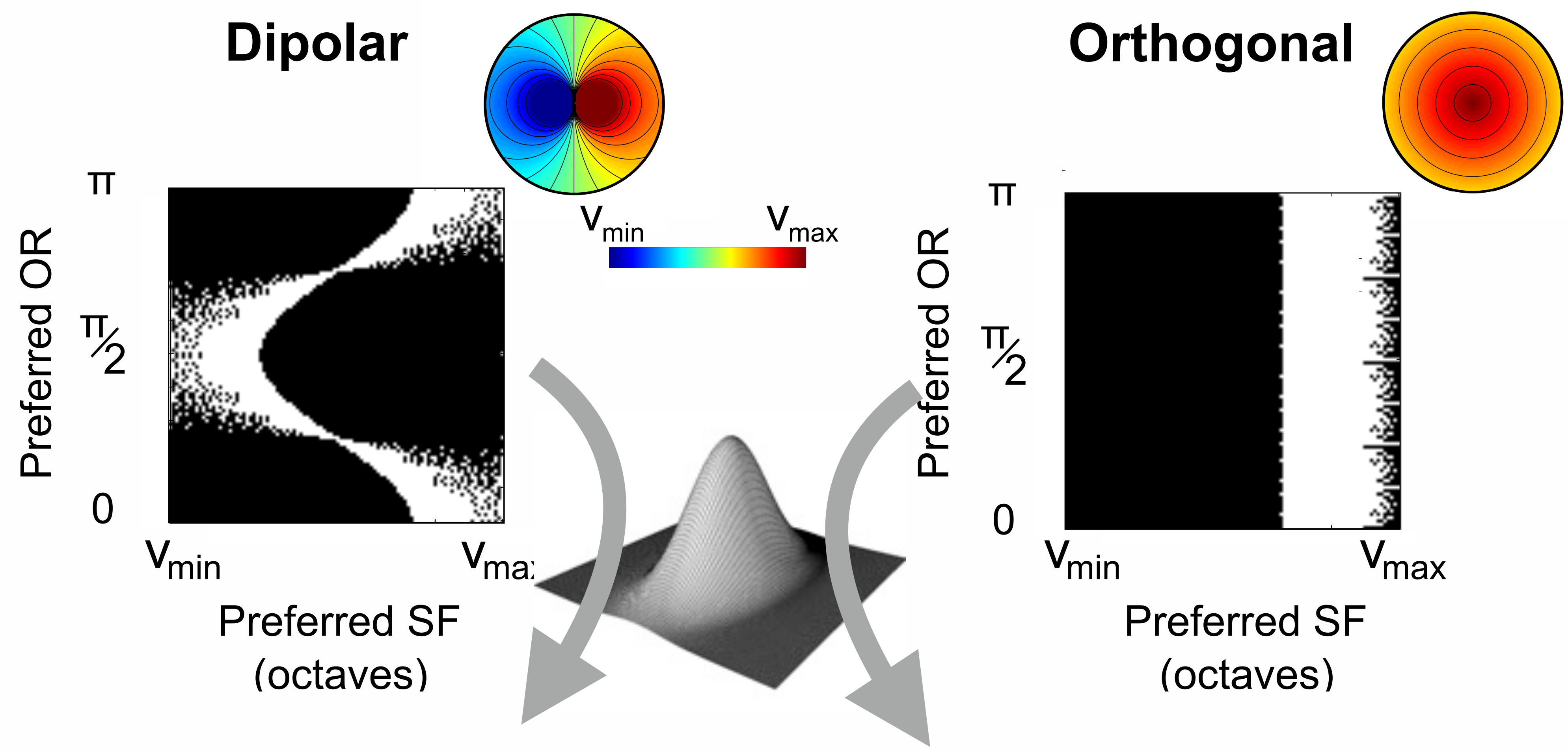
Coding properties of Pinwheel-Dipoles

Preferred OR/SF space

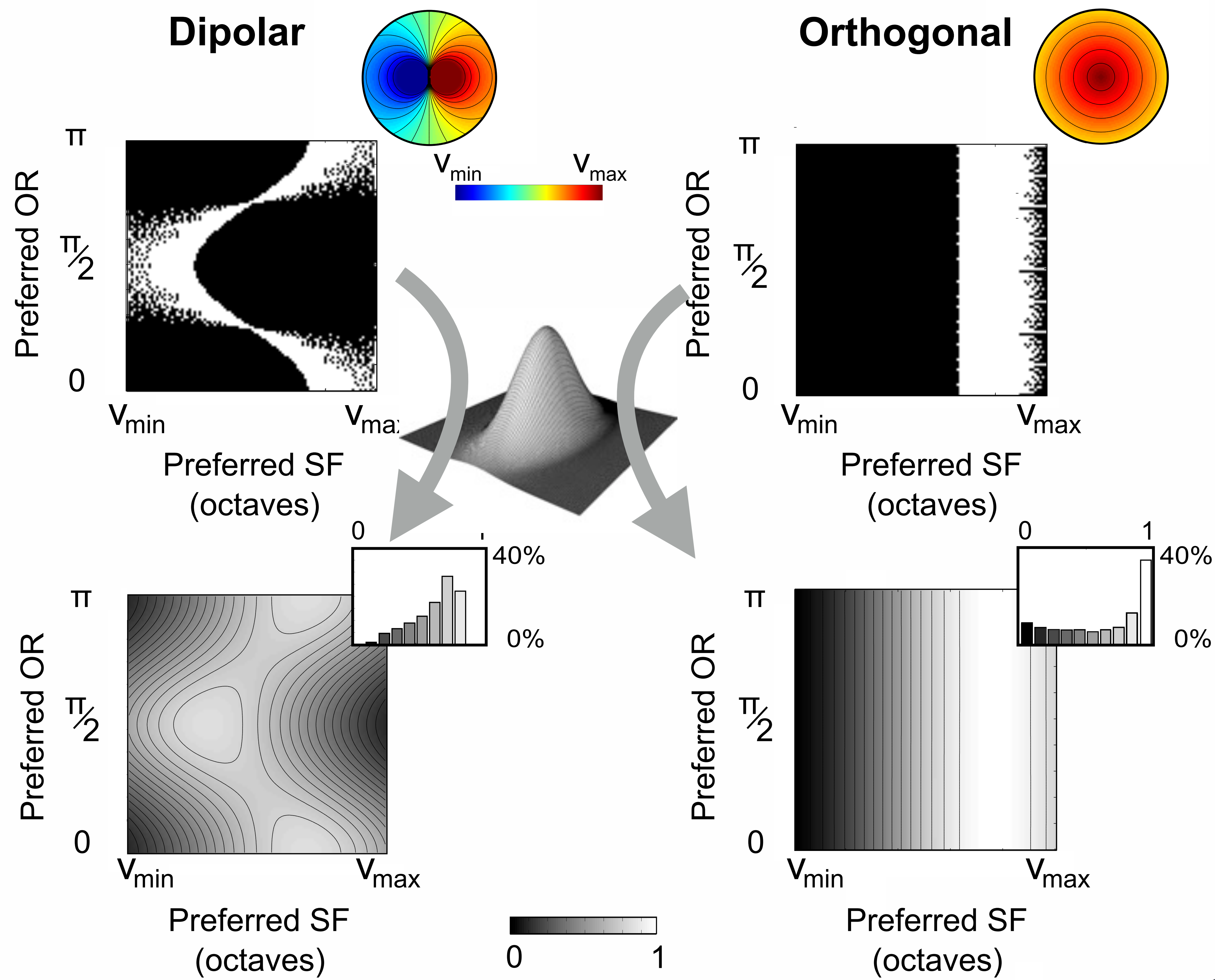




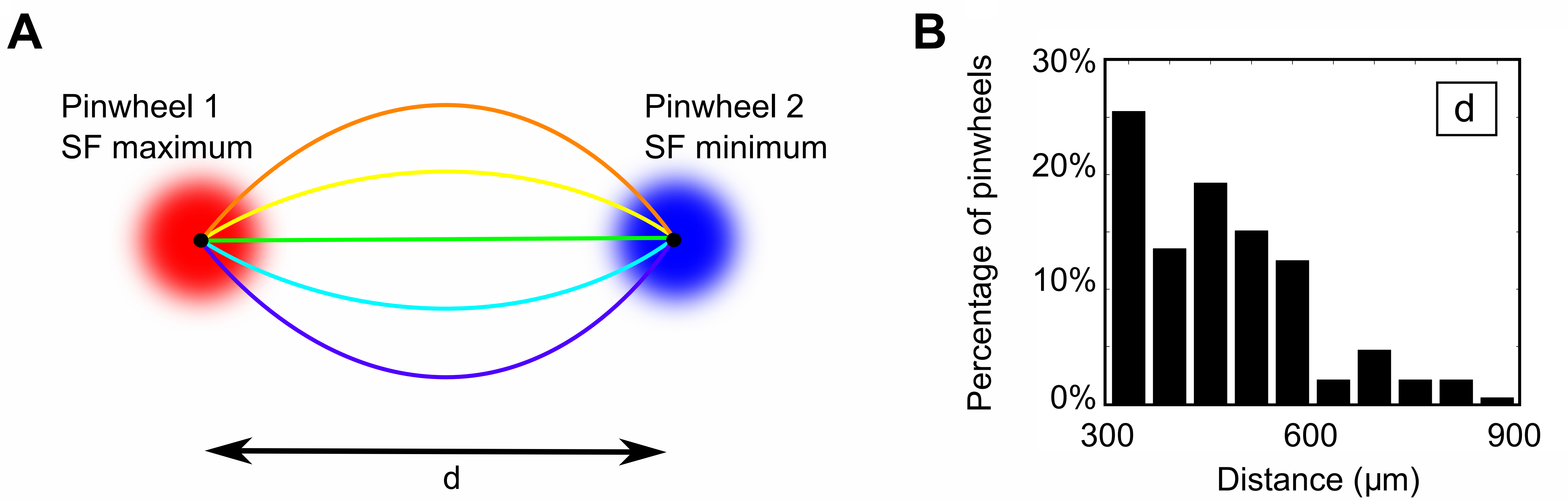
Preferred OR/SF space



Preferred OR/SF space



Fit of the putative orthogonal model

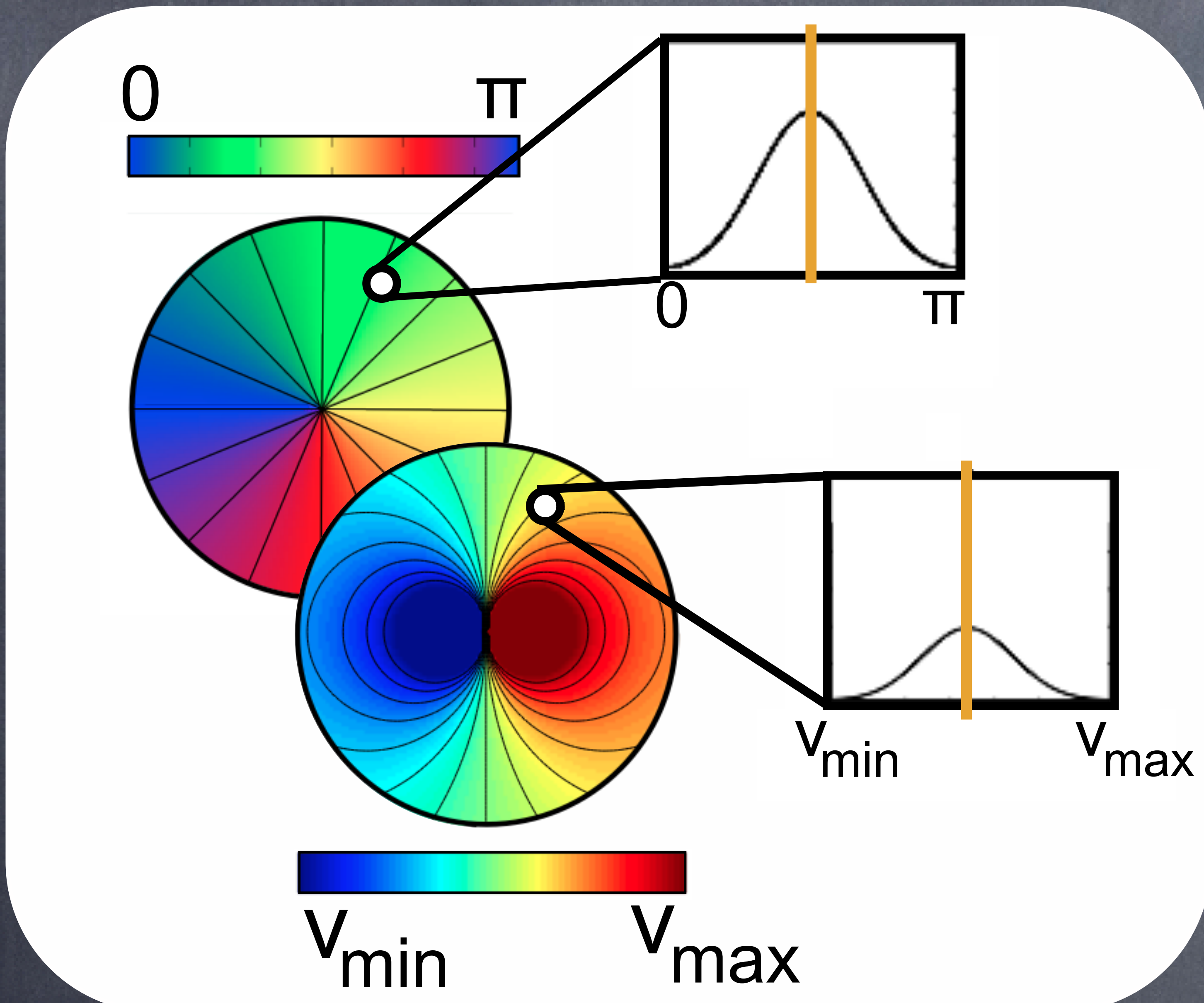


$$\nu_{pol}(r,\phi) = \nu_{max} - \beta\, r$$

$$\beta = \Delta\nu_{extr}/d$$

Coding/Decoding algorithm

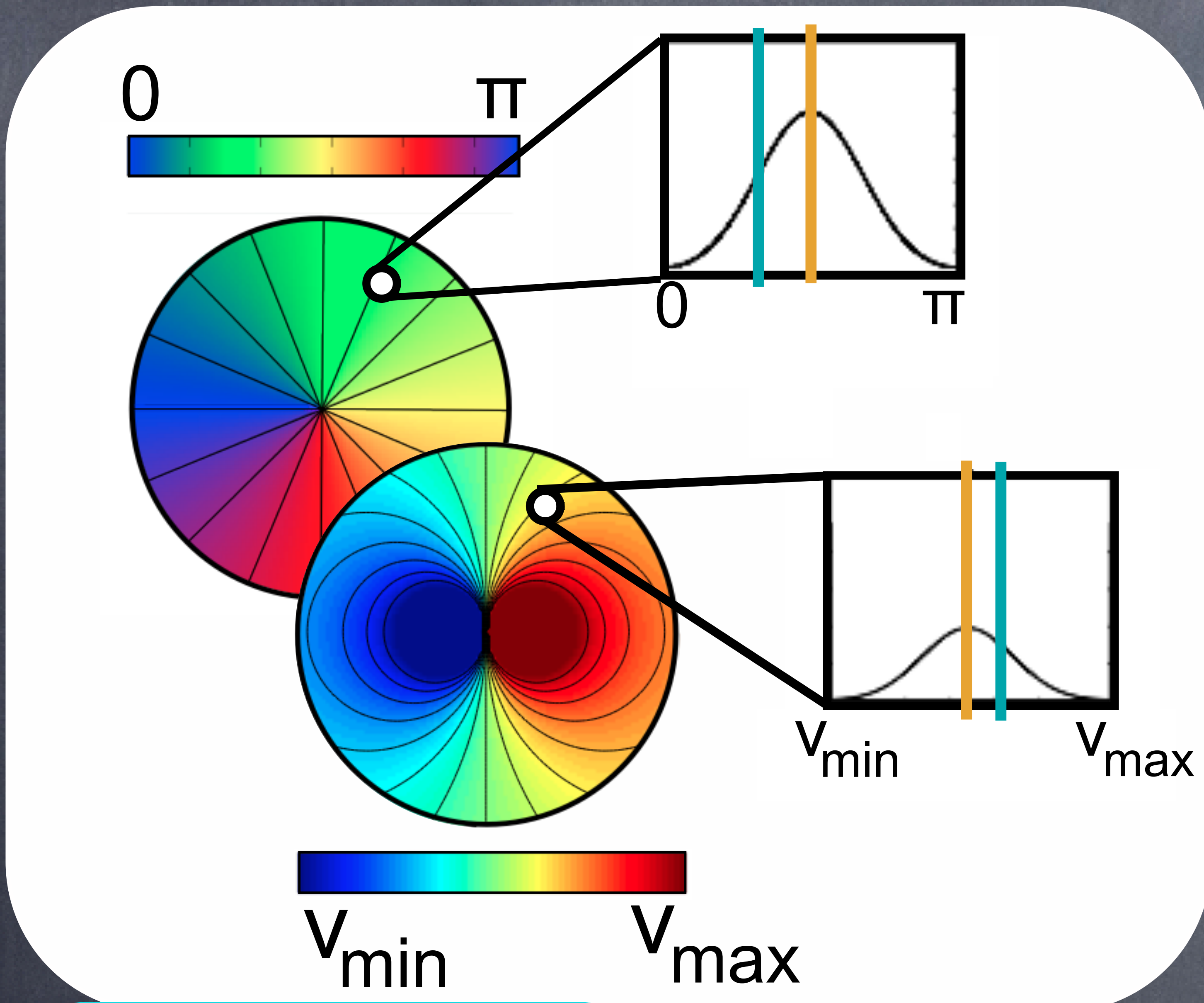
Local Detection

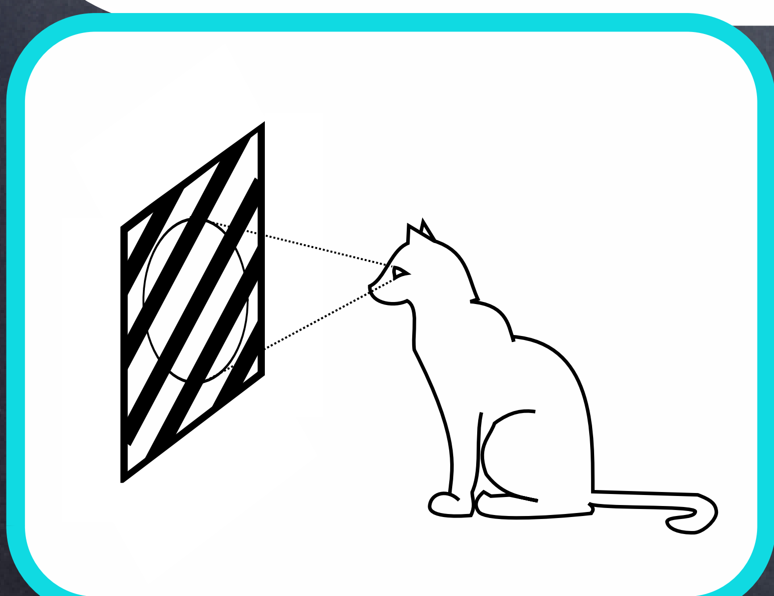


$\{OR_{pref}, SF_{pref}\}$

Coding/Decoding algorithm

Local Detection



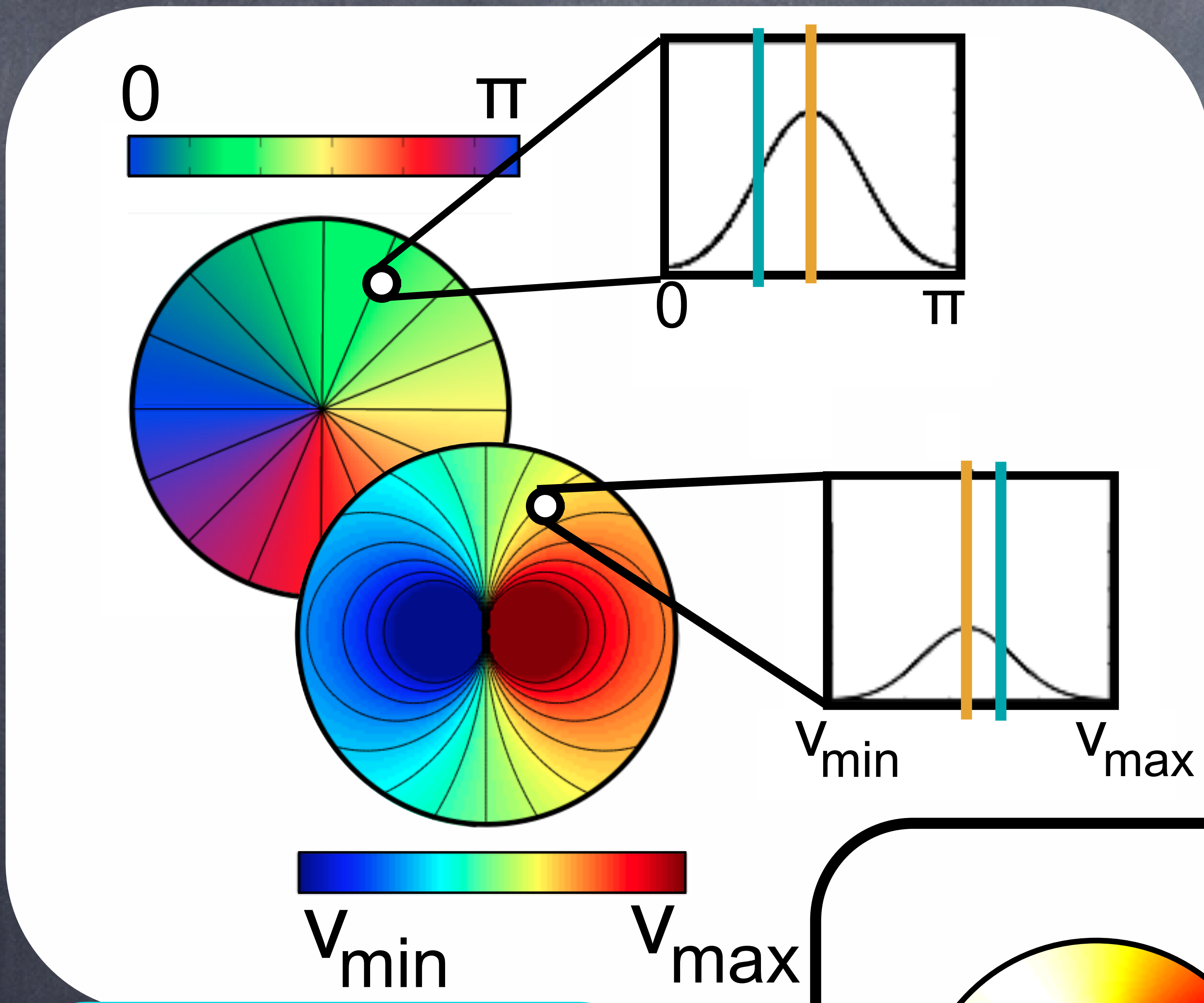


$\{OR_{pref}, SF_{pref}\}$

$\{OR_{stim}, SF_{stim}\}$

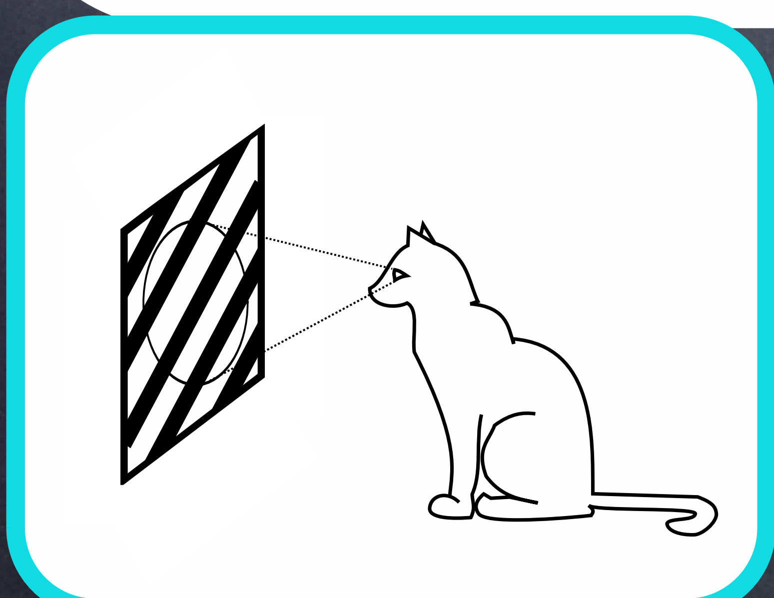
Coding/Decoding algorithm

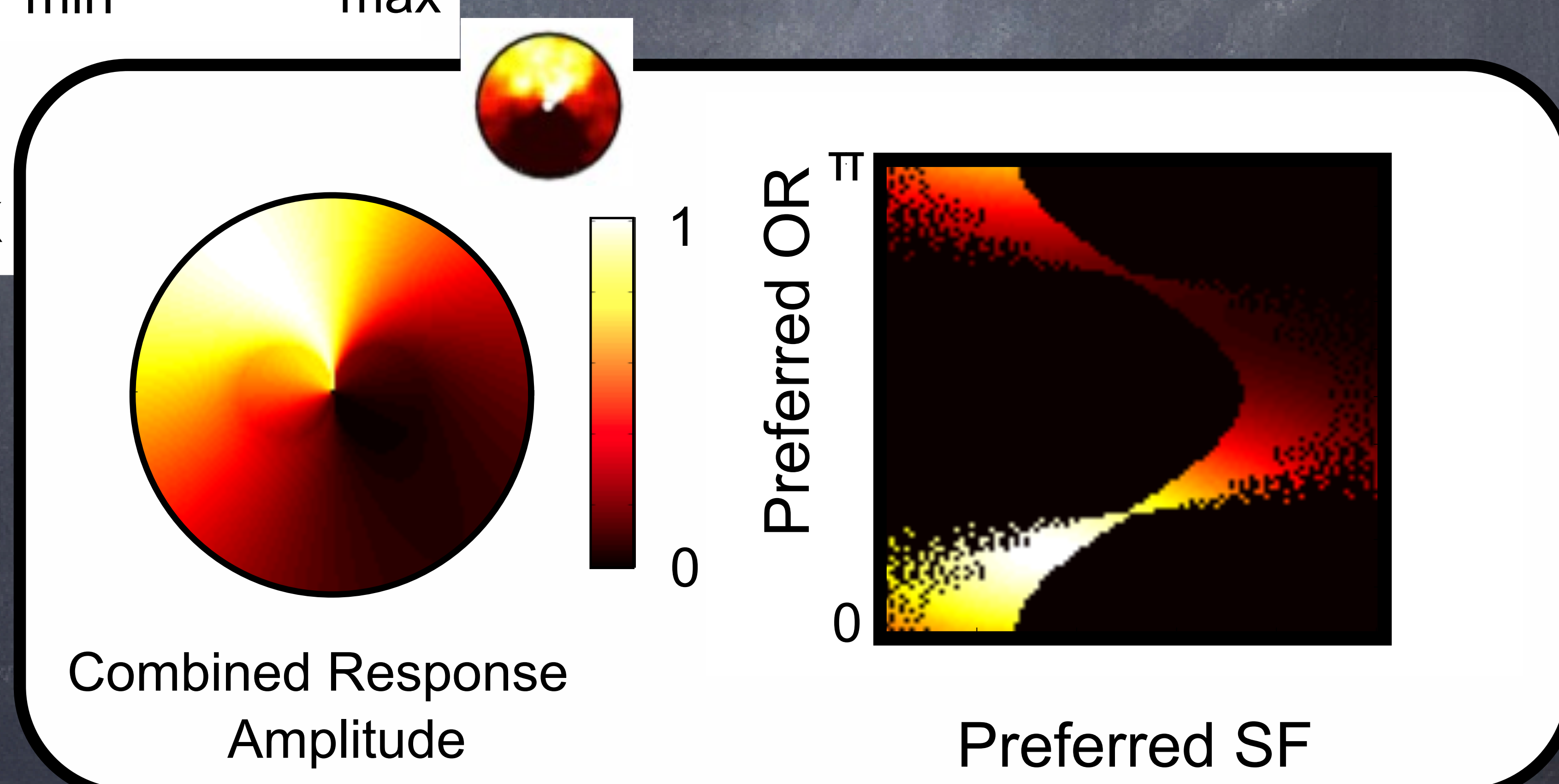
Local Detection



$\{OR_{pref}, SF_{pref}\}$

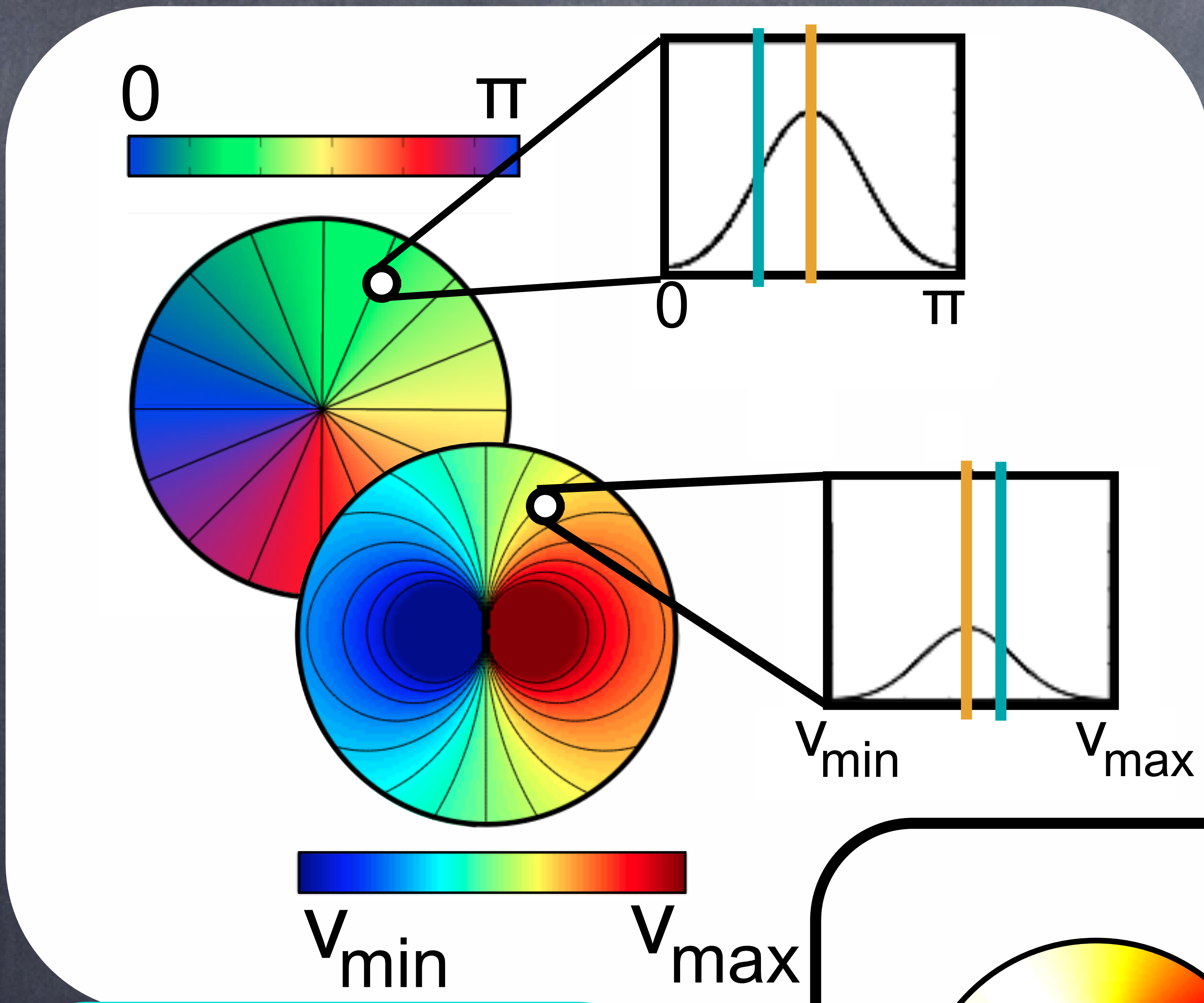
$\{OR_{stim}, SF_{stim}\}$





Coding/Decoding algorithm

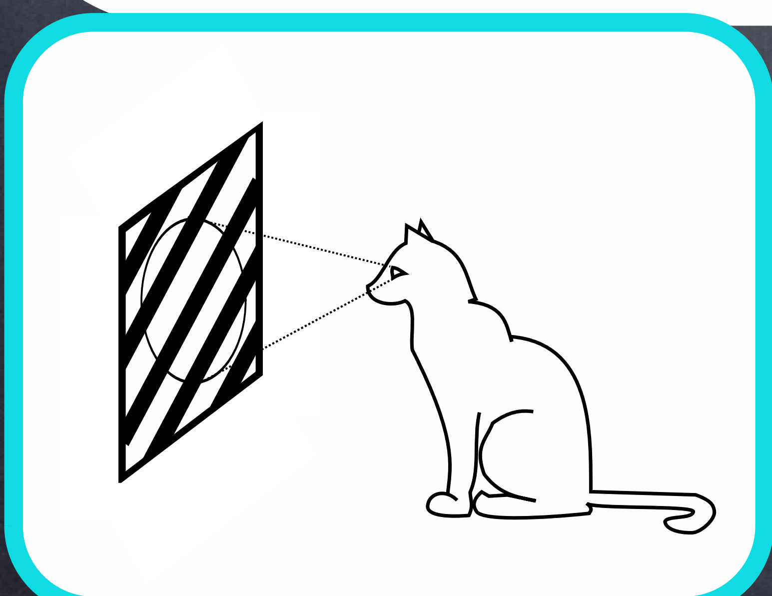
Local Detection

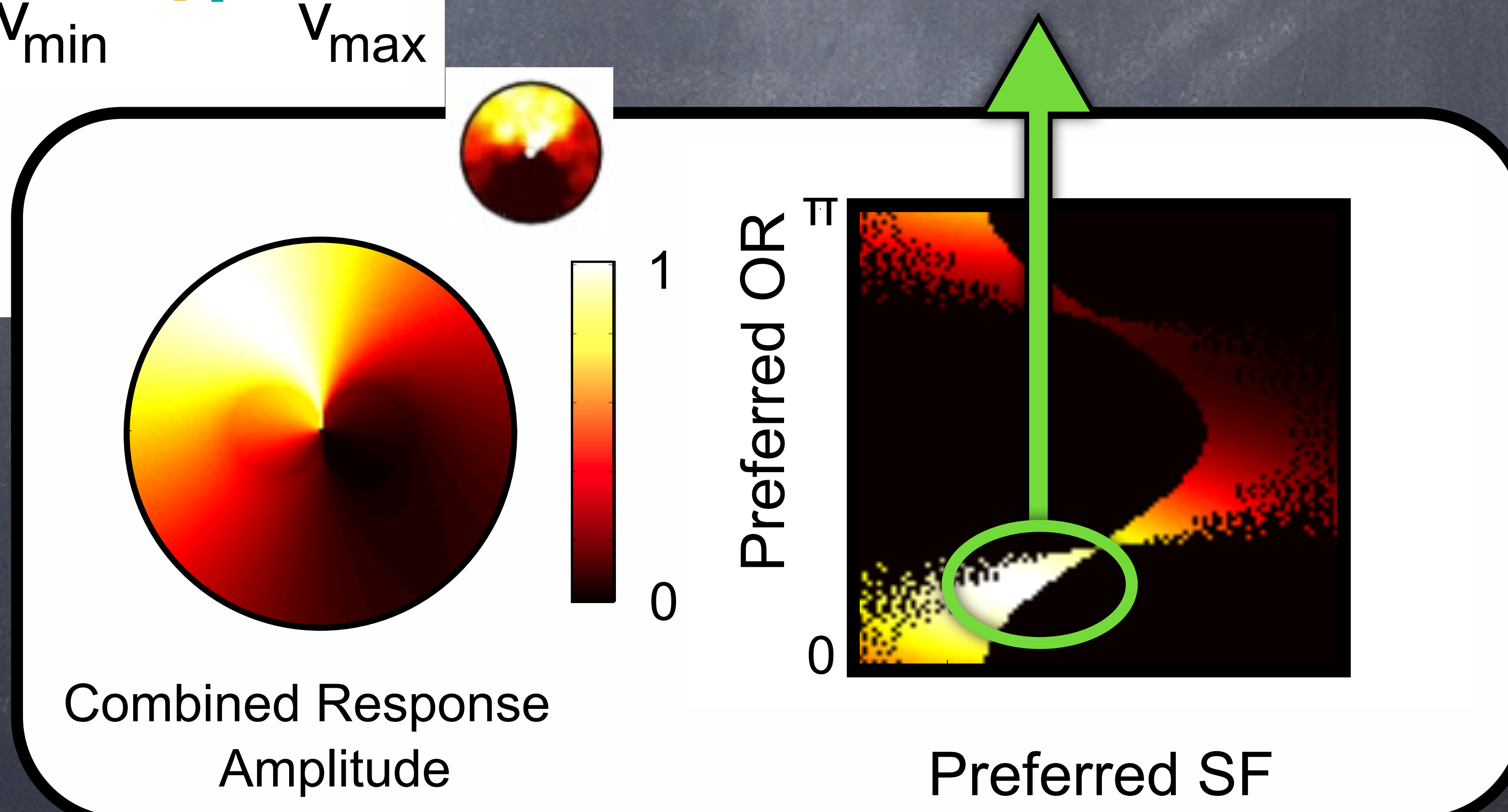


$\{OR_{pref}, SF_{pref}\}$

$\{OR_{stim}, SF_{stim}\}$

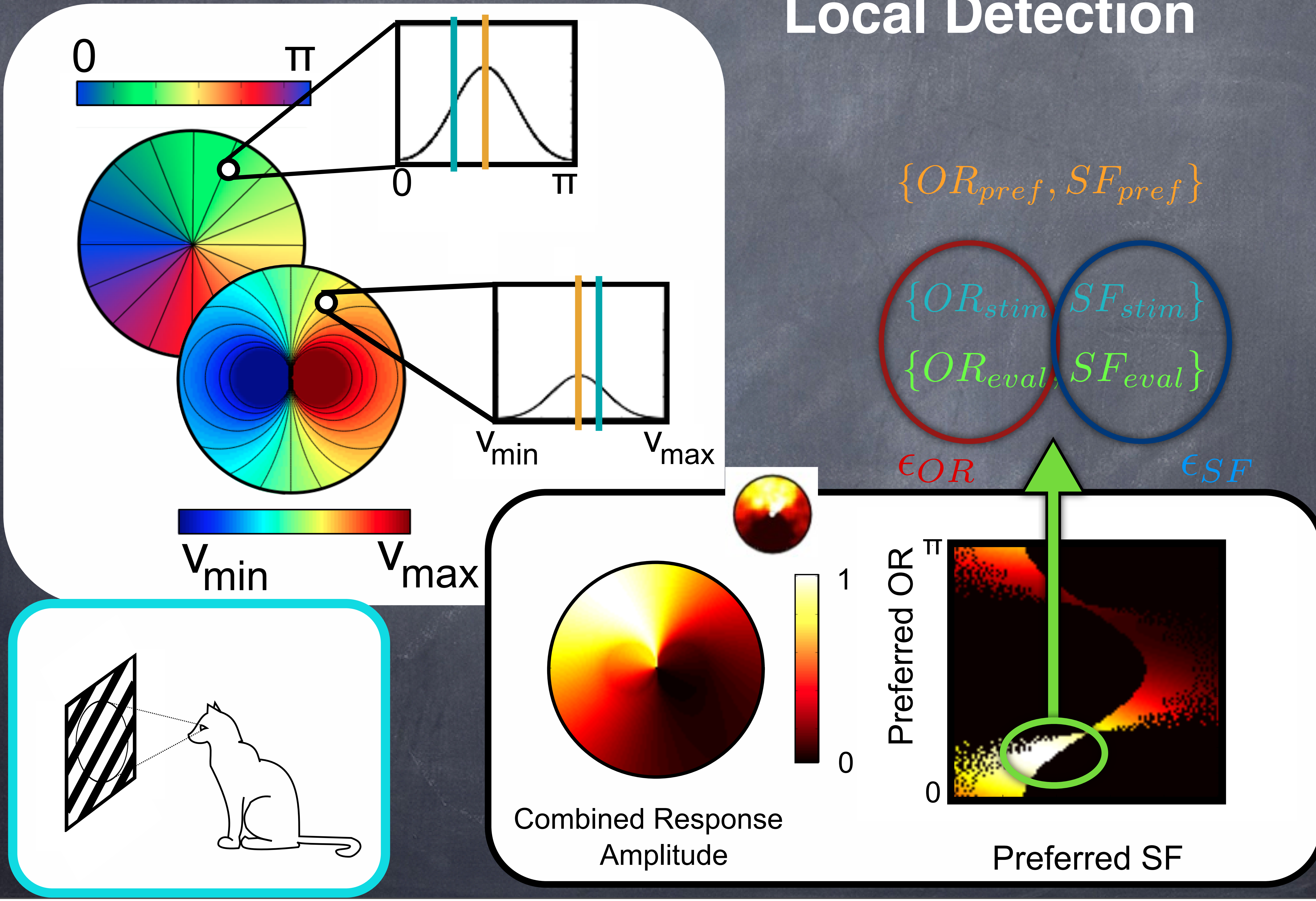
$\{OR_{eval}, SF_{eval}\}$



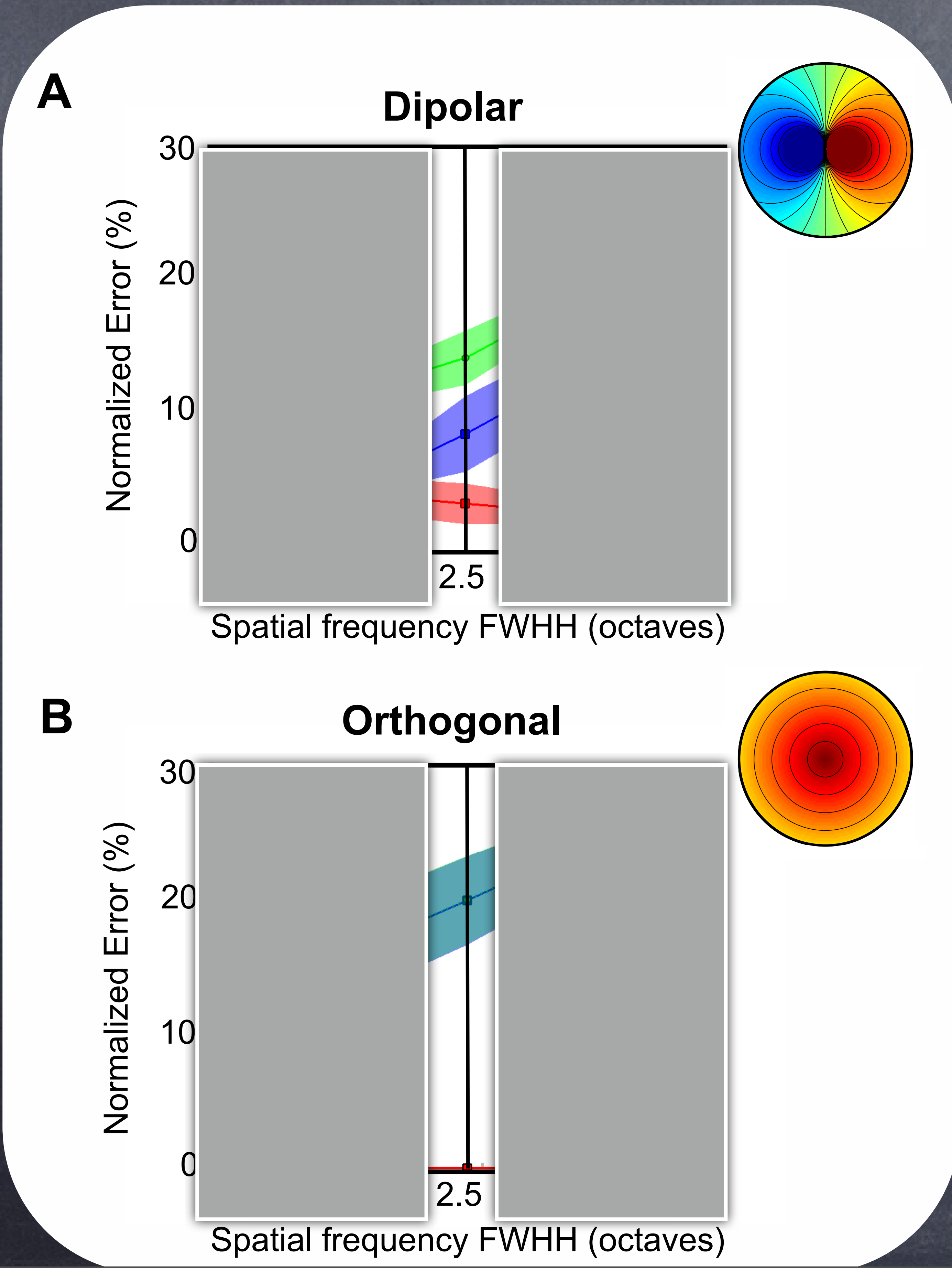


Coding/Decoding algorithm

Local Detection

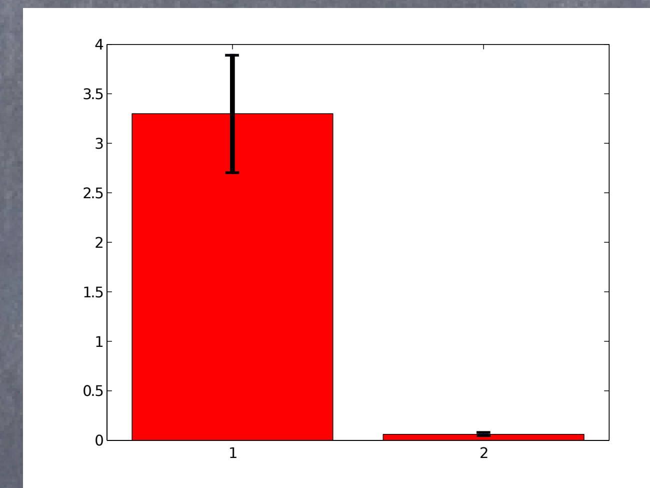


A “prediction” of the model: Trade-off

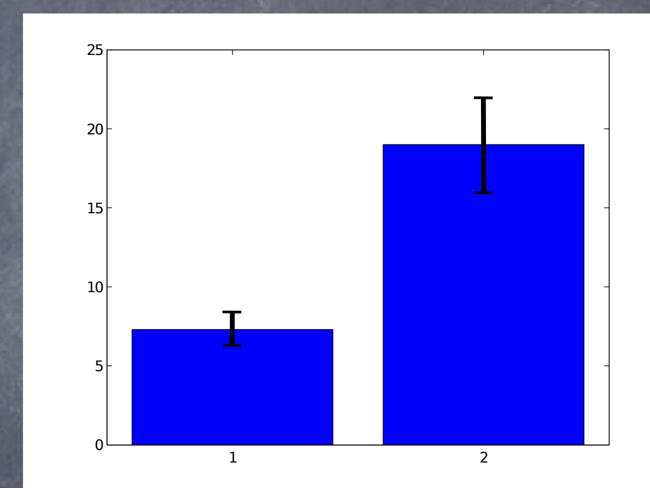


Dip Orth

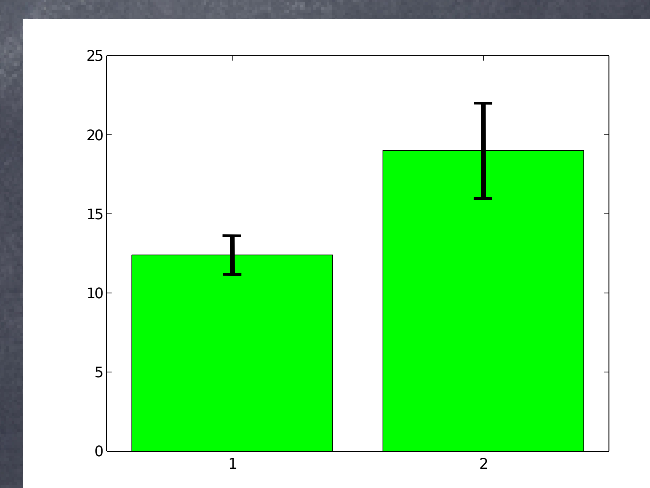
OR



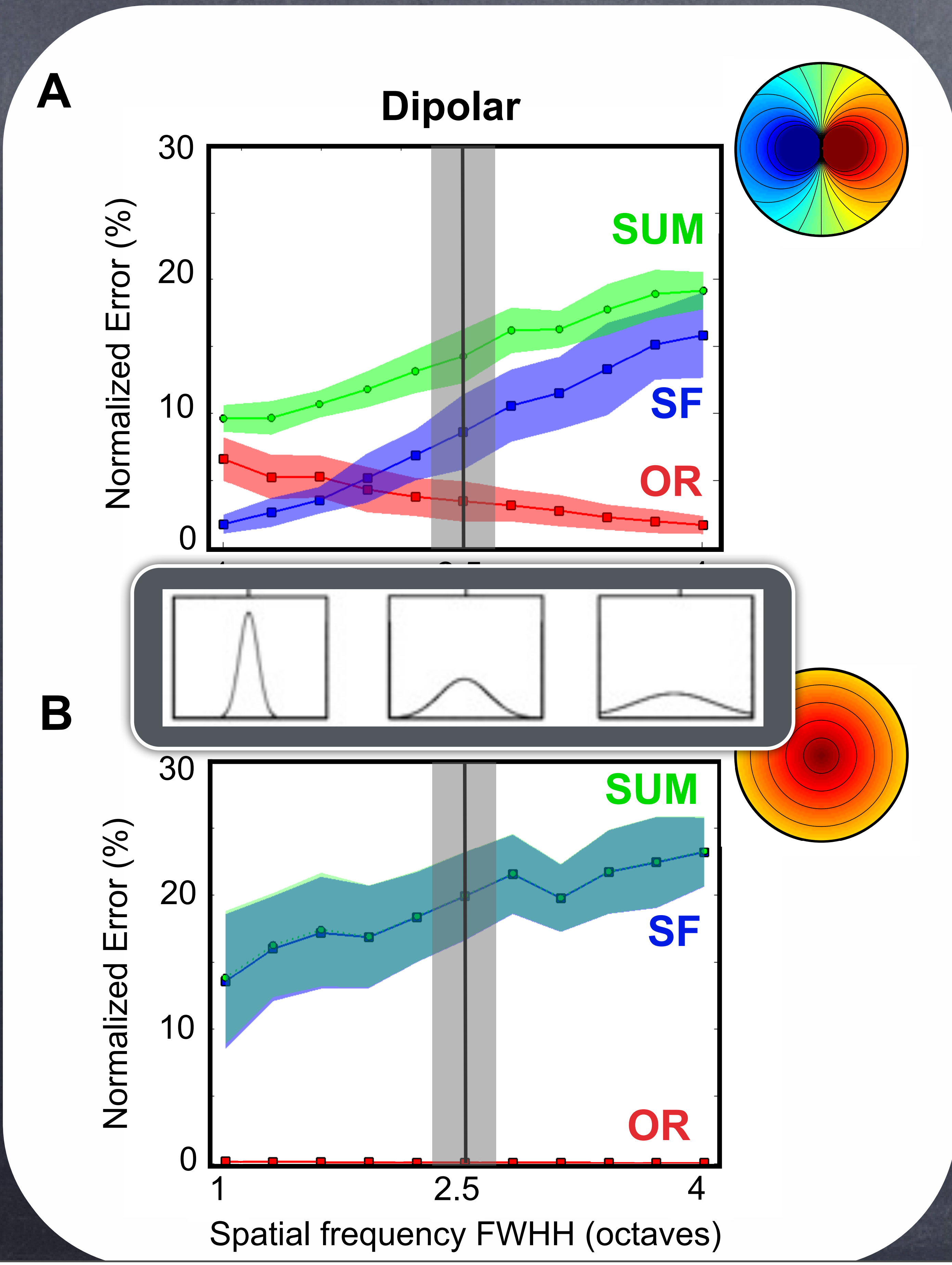
SF

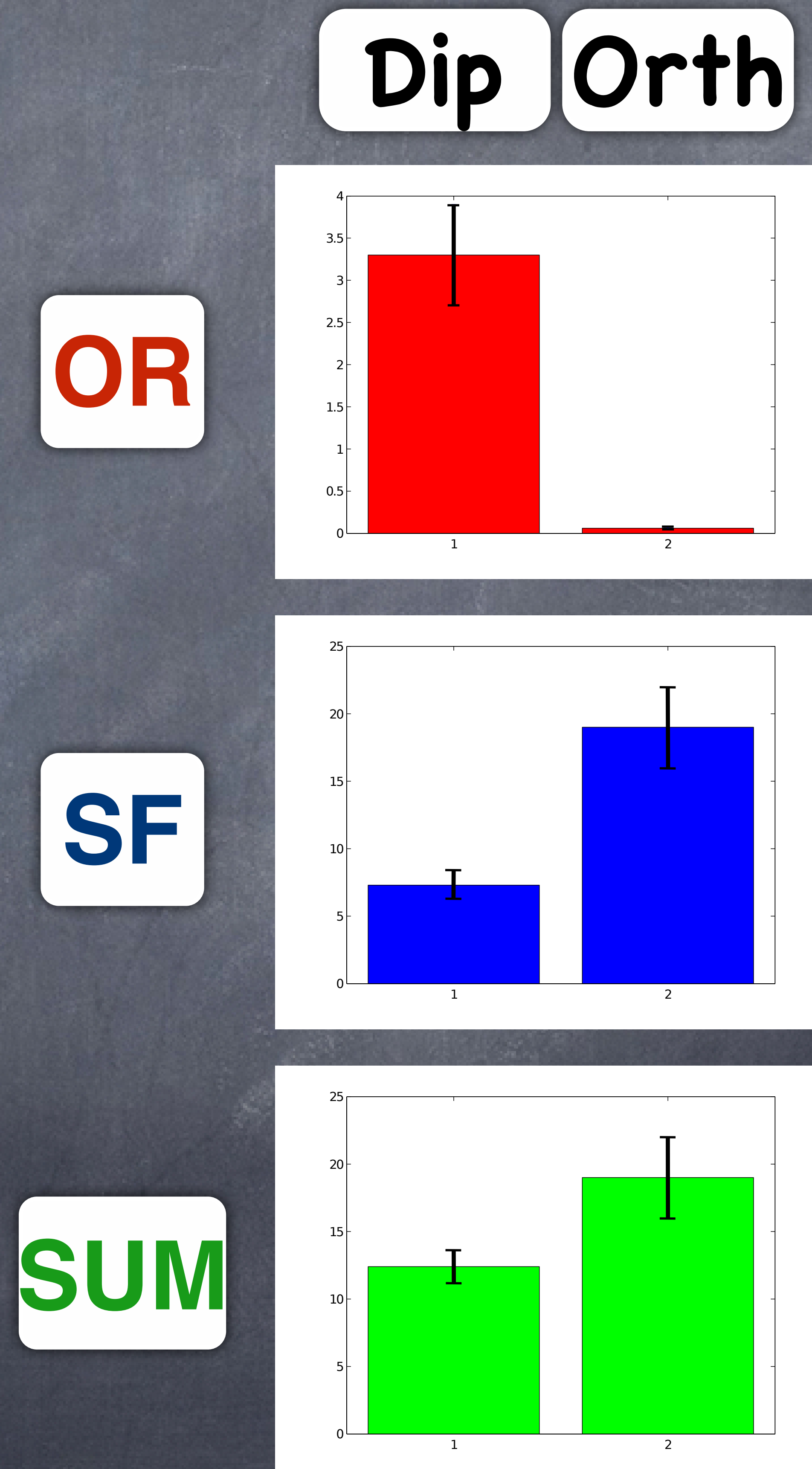


SUM

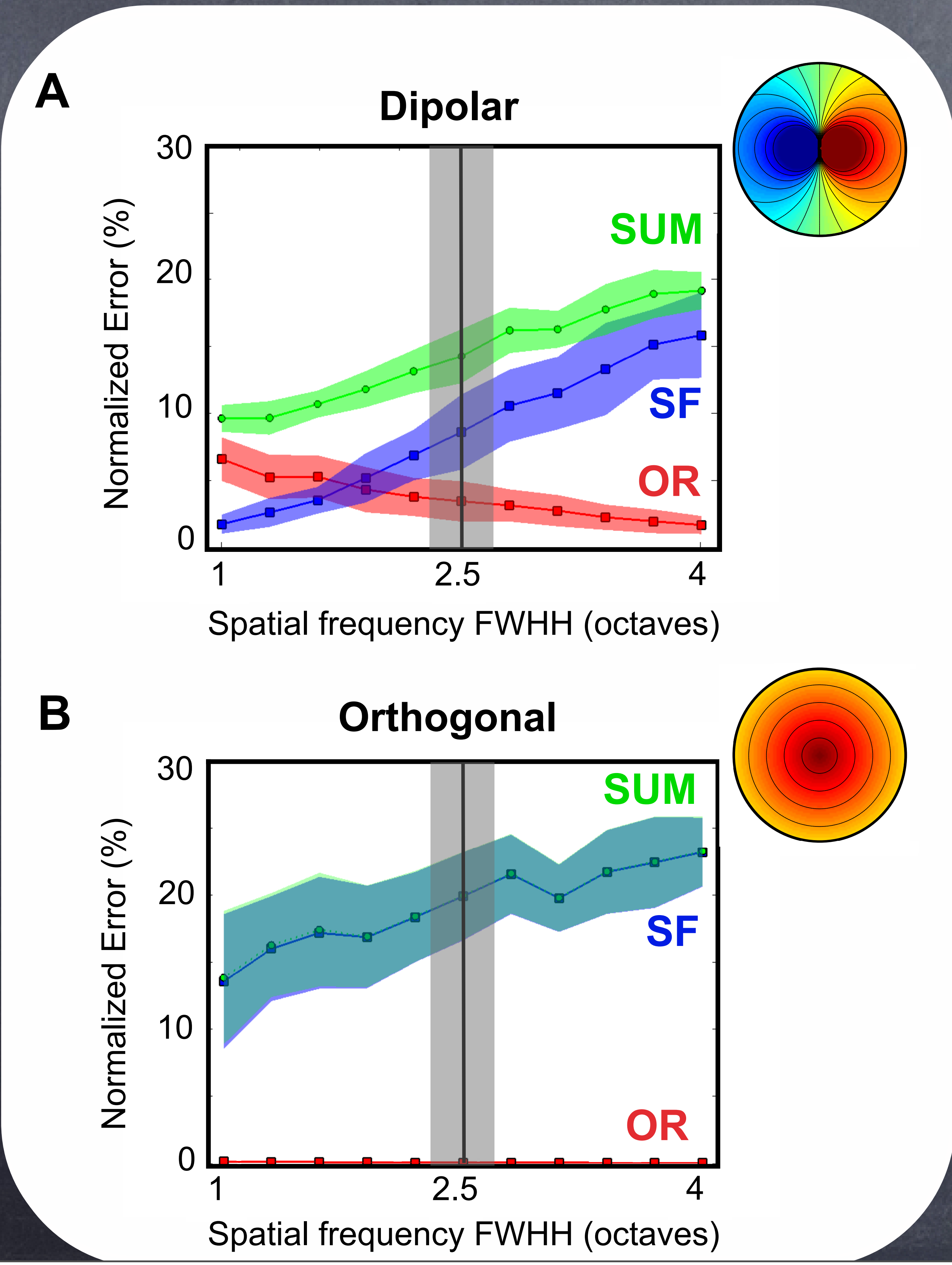


A “prediction” of the model: Trade-off





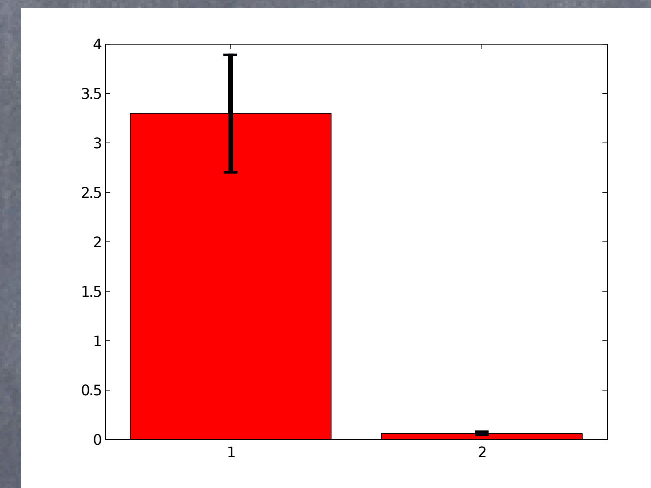
A “prediction” of the model: Trade-off



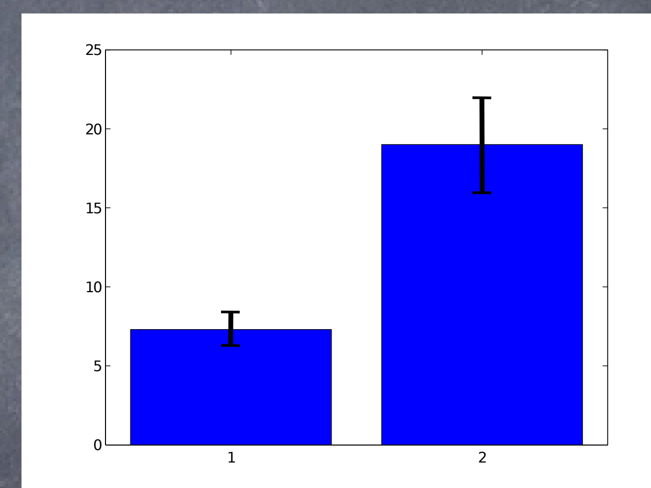
Dip

Orth

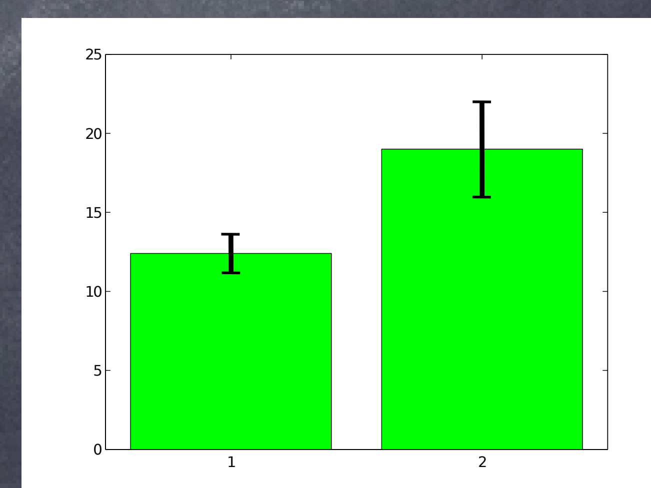
OR



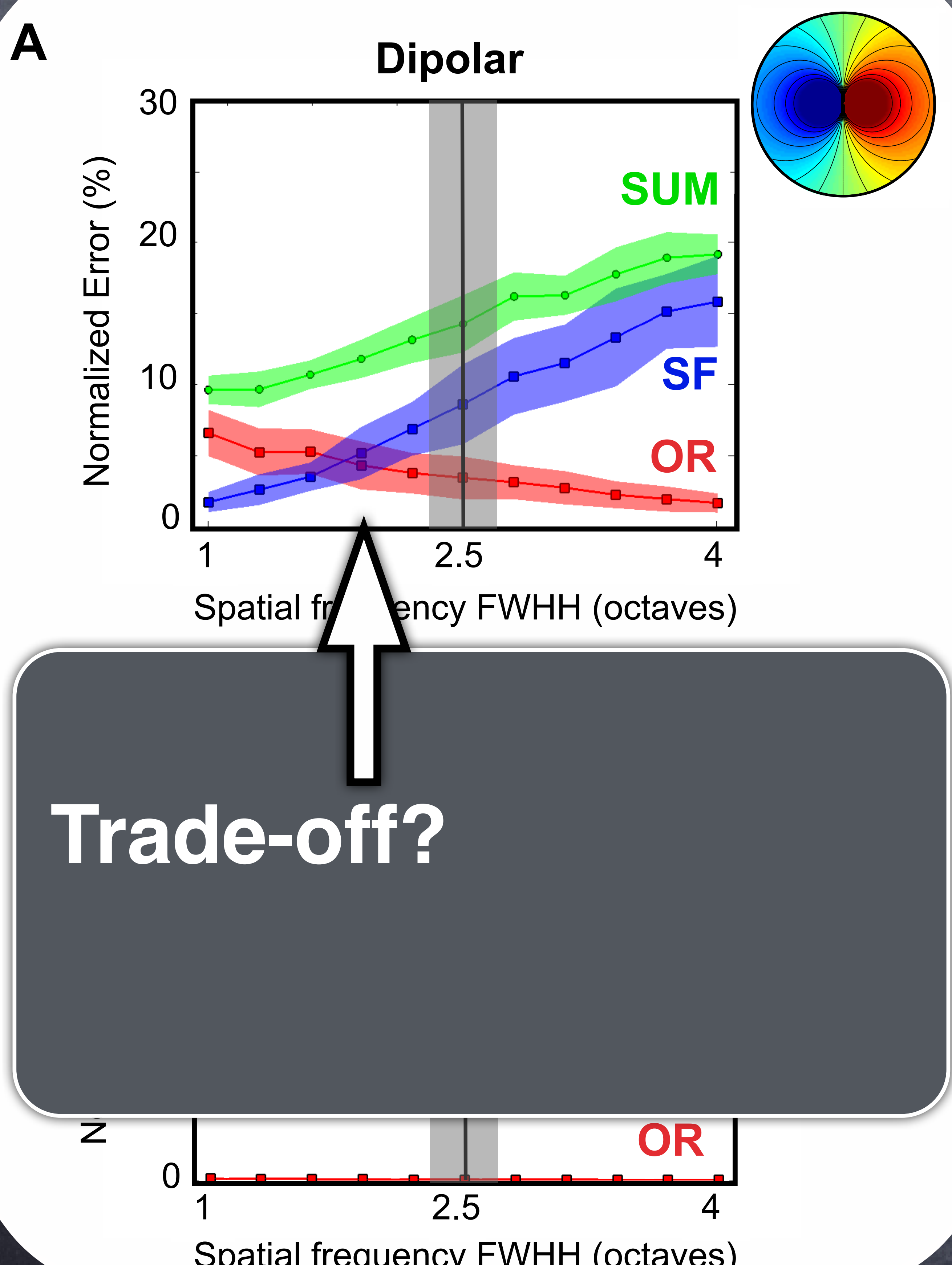
SF



SUM



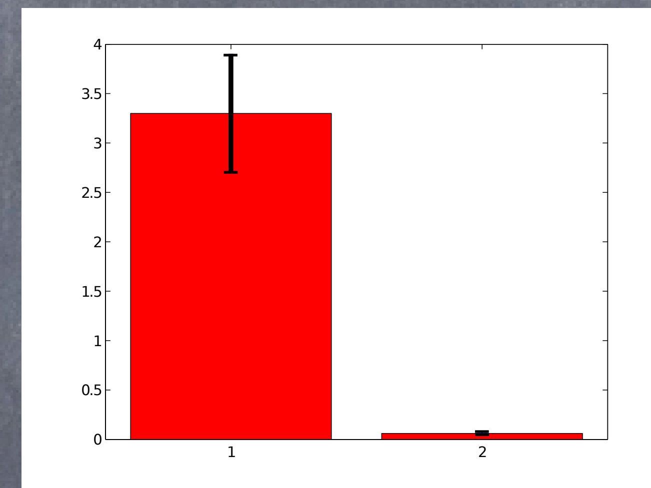
A “prediction” of the model: Trade-off



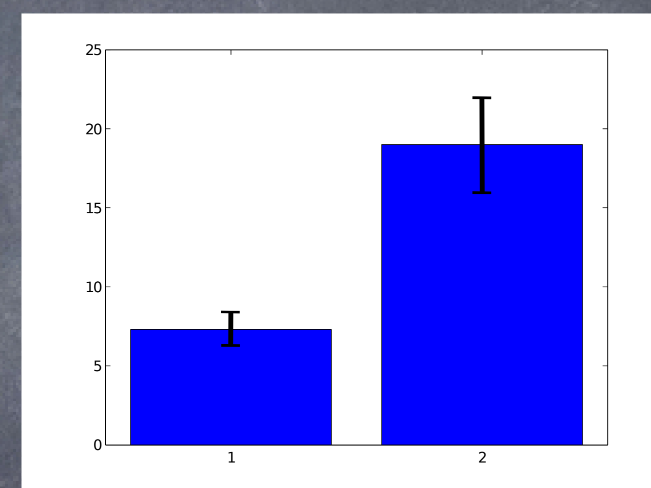
Trade-off?

Dip | Orth

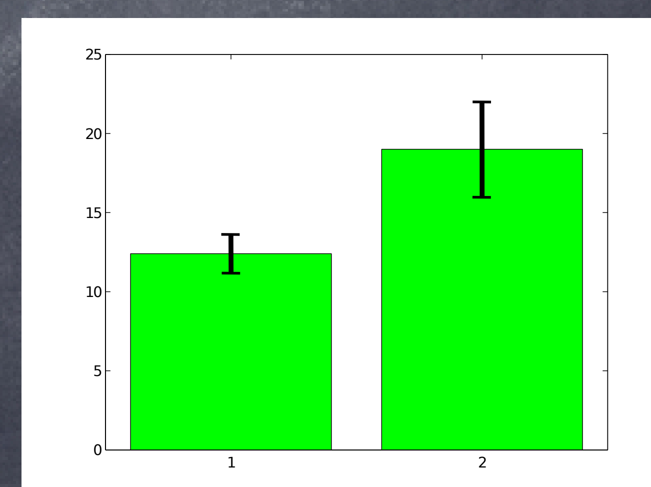
OR



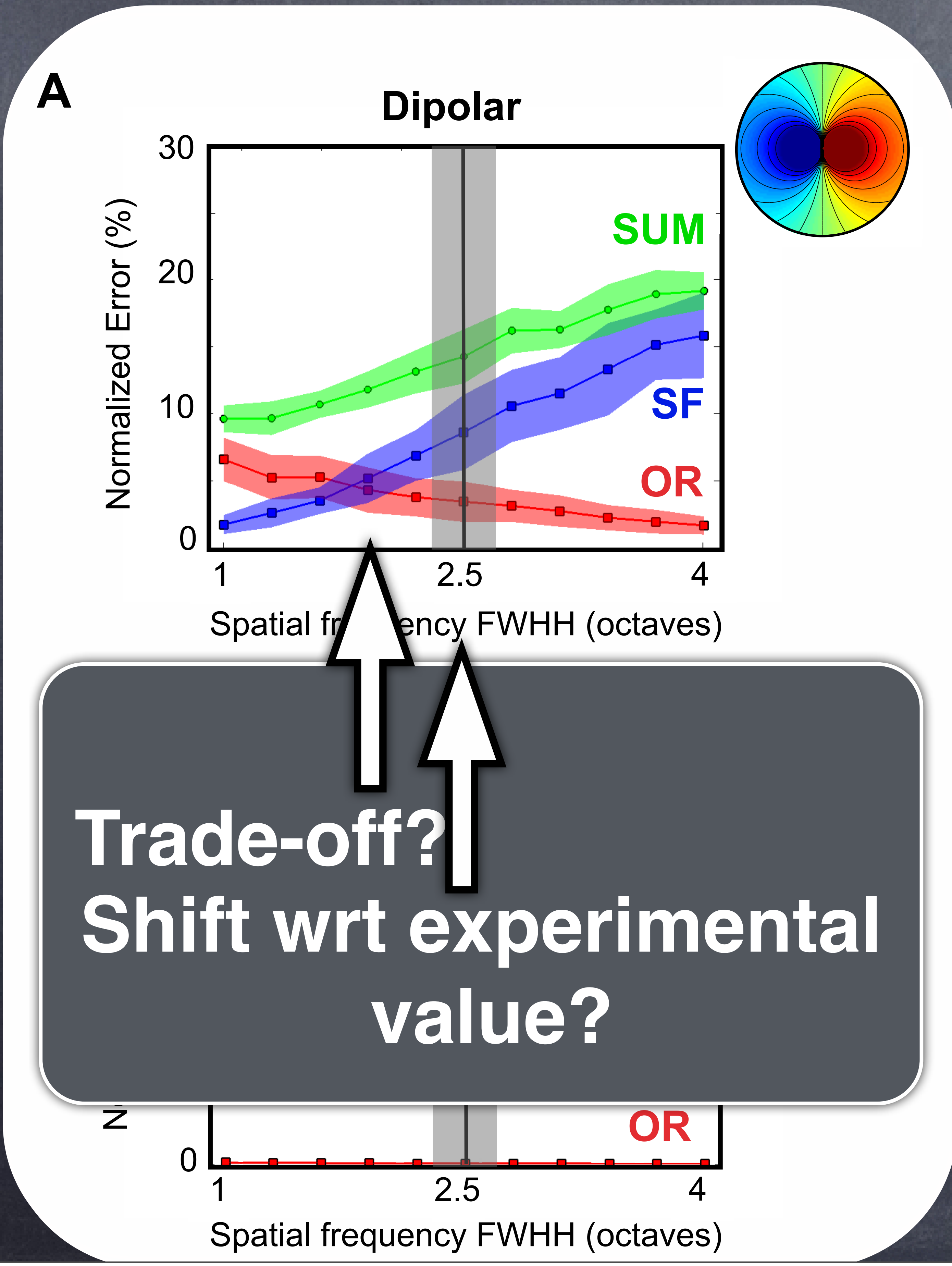
SF



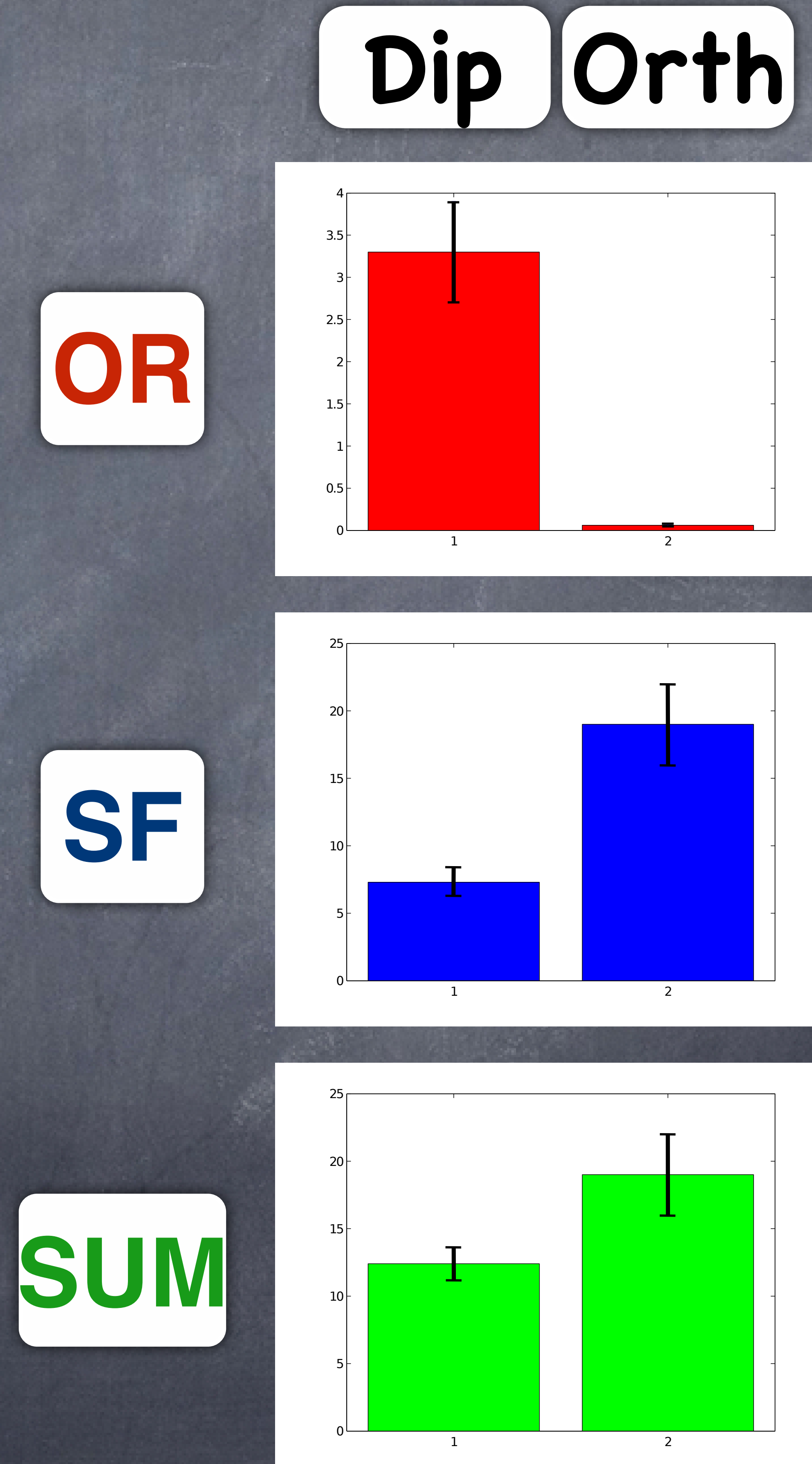
SUM



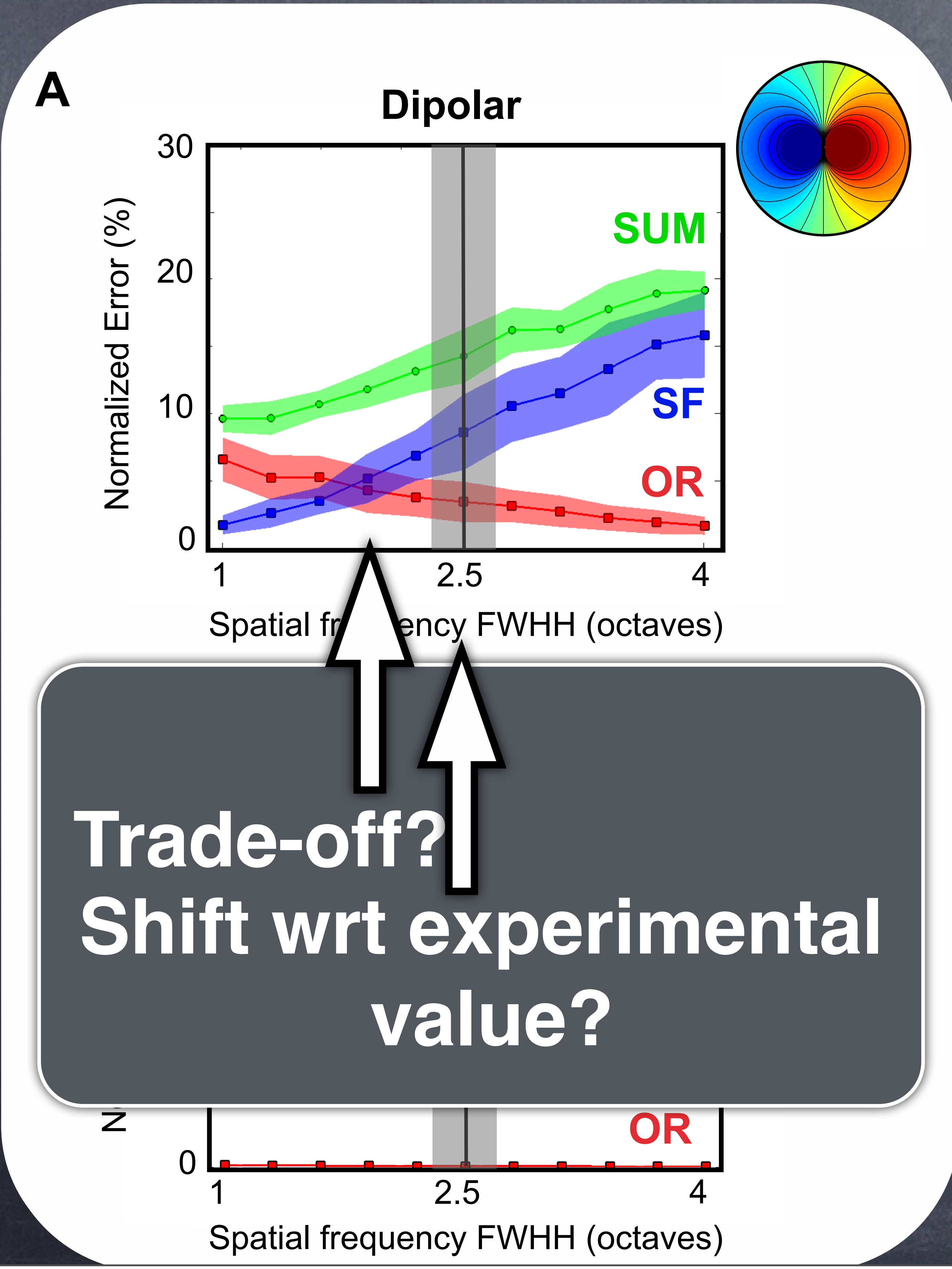
A “prediction” of the model: Trade-off



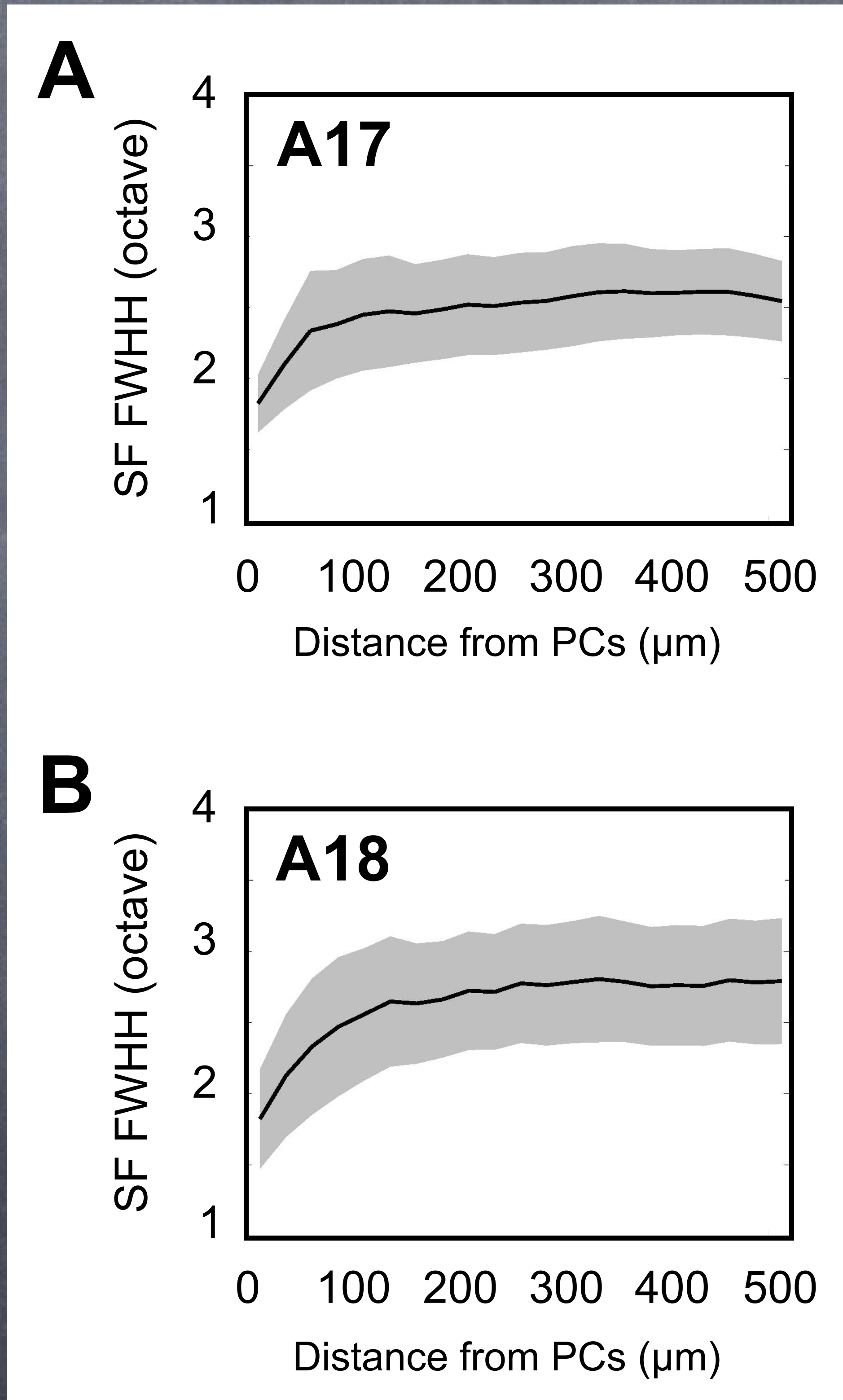
Trade-off?
Shift wrt experimental value?



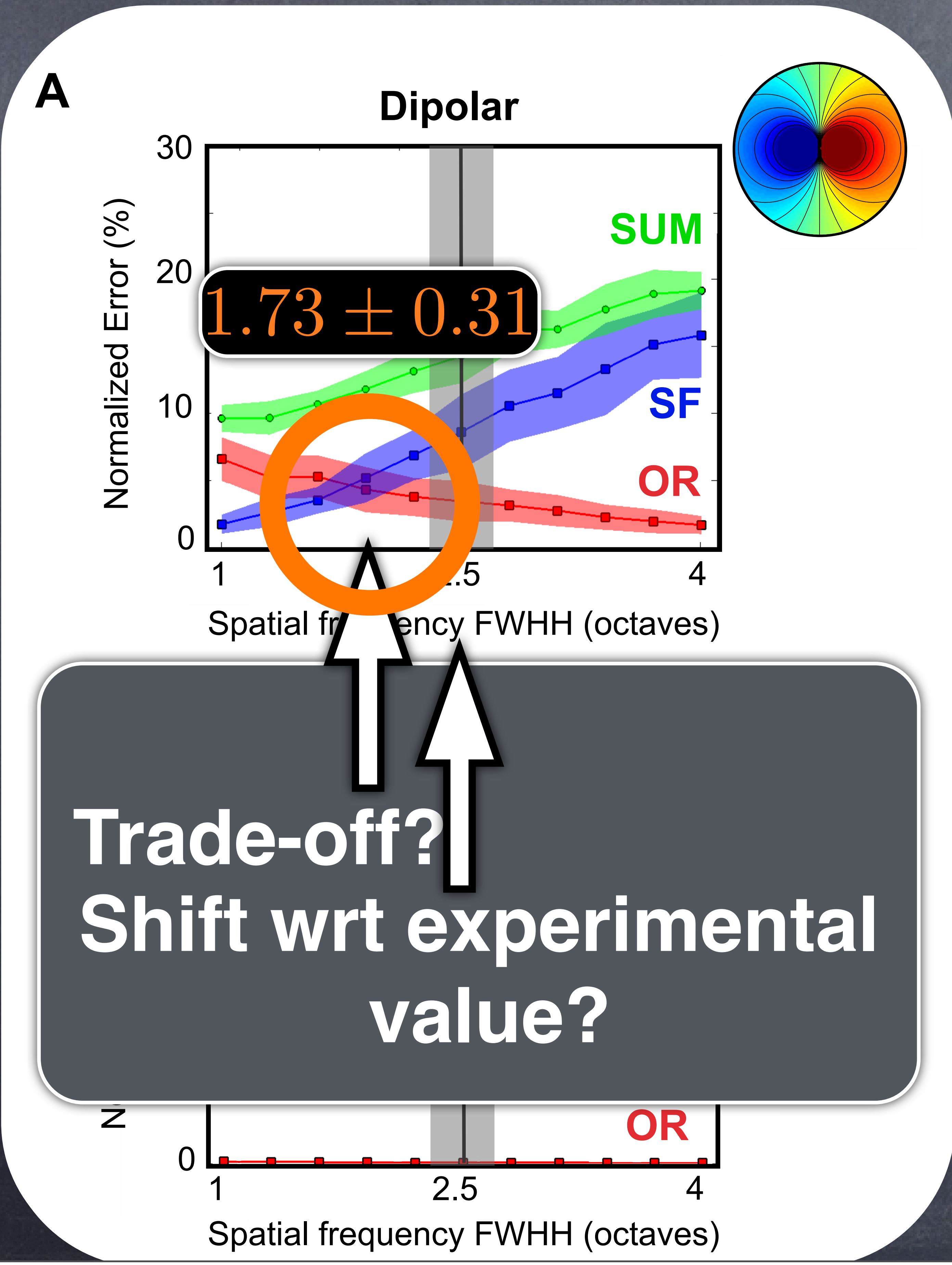
A “prediction” of the model: Trade-off



Trade-off?
Shift wrt experimental value?

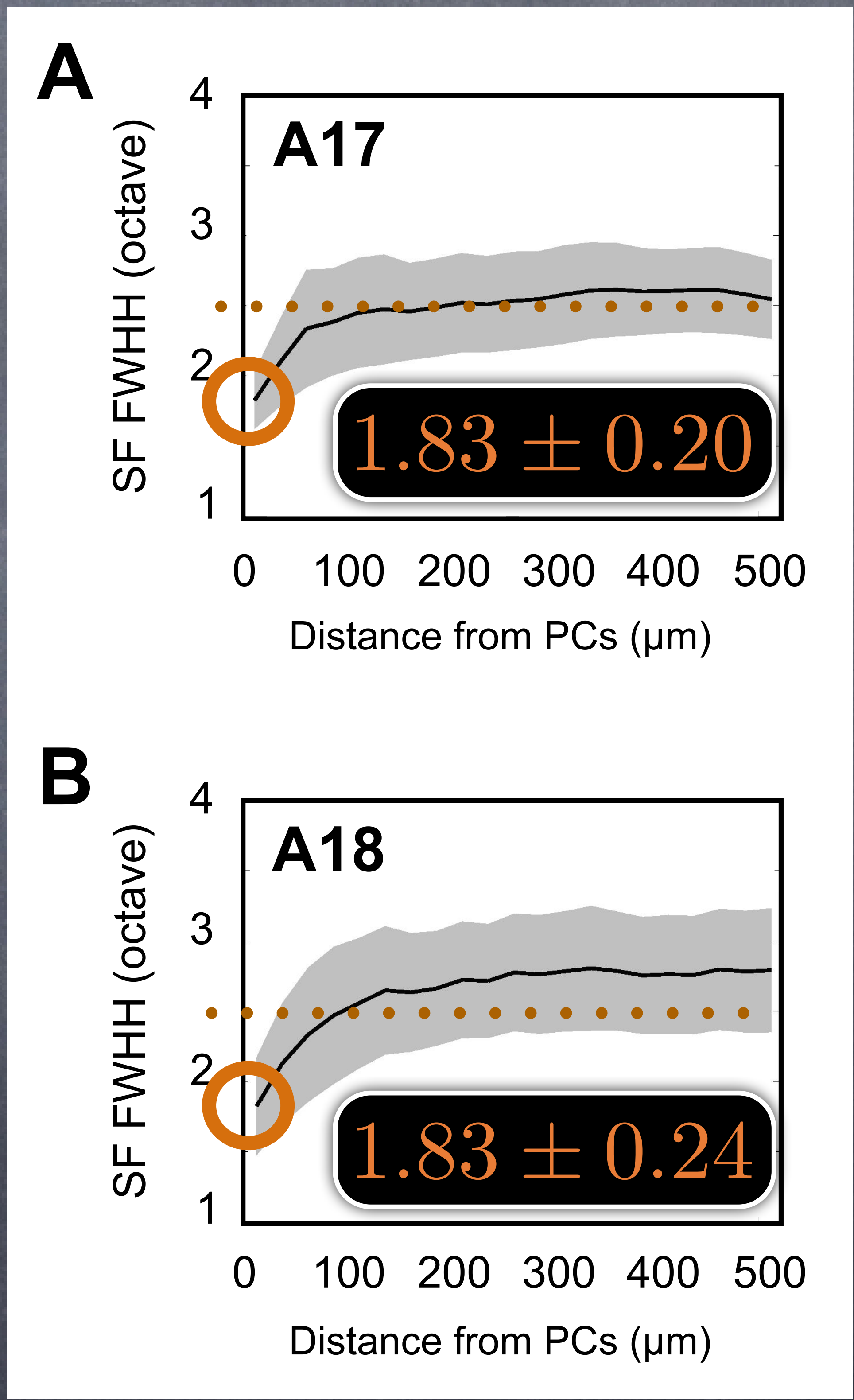


A "prediction" of the model: Trade-off



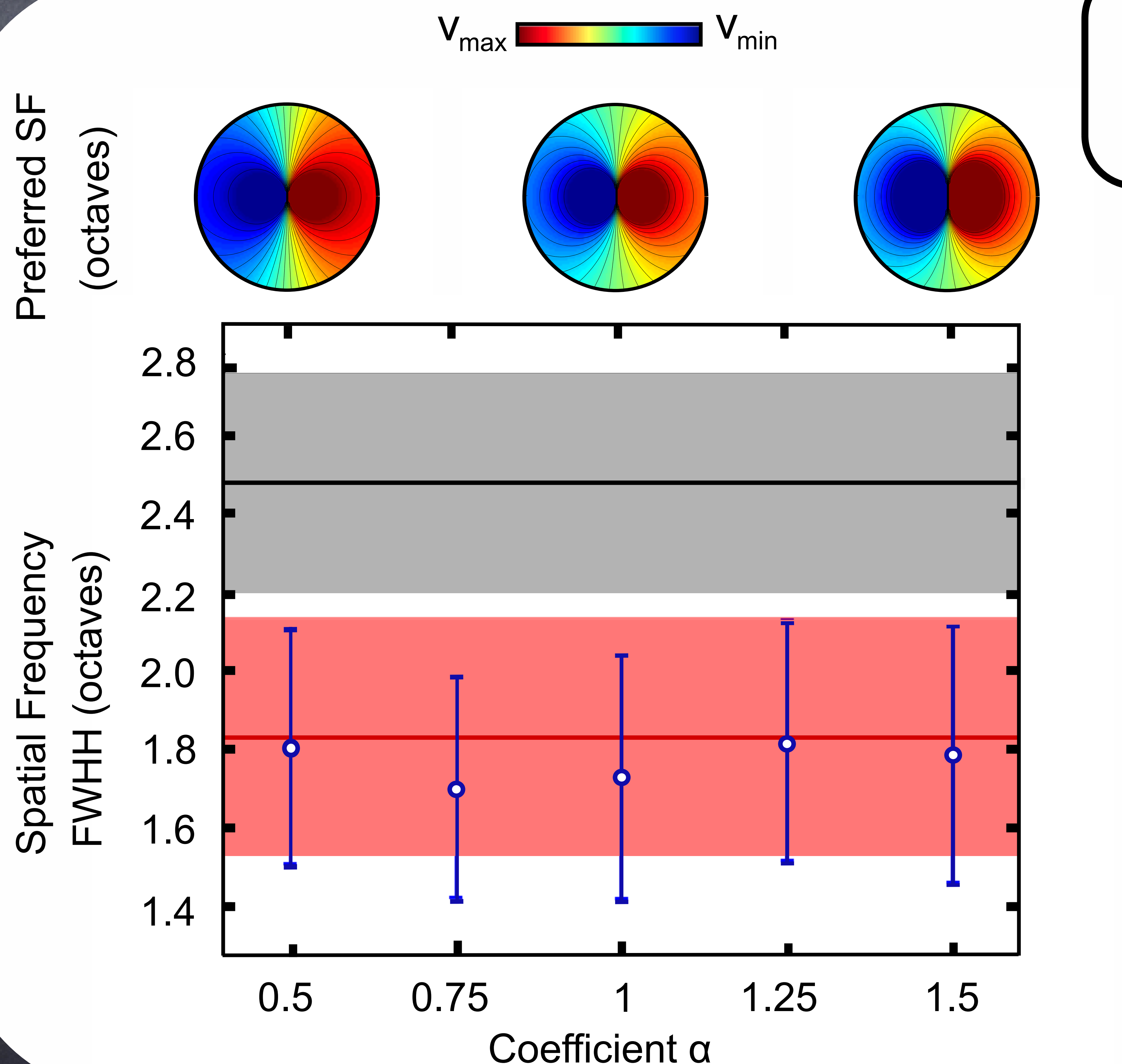
Trade-off?

Shift wrt experimental value?



Robustness of balanced detection paradigm

$$SF \sim \frac{\cos\varphi}{r^{\alpha}}$$



Conclusion

show singularities co-localized with the pinwheels

- SF map can be locally modeled by dipoles

- General organizing principles (exhaustivity and parsimony) seem to be involved

- Pinwheel-Dipoles architectures show comparable (or better) efficient local coding properties than uniform coverage

- A trade-off can explain the sharpening of tuning widths near pinwheel centers found experimentally

Open questions:

- How and why are these results species dependent ?

- Why different singularities are co-localized?
  Das and Gilbert, Nature 1997

- If "local is good", why not more pinwheels? Is there another trade-off in the game? Interaction with other principles (continuity/coverage)?
  Keil and Wolf, Neural Systems & Circuits (2011);
  Reichl et al., (I and II), PLOS Comp. Bio. (1990)

- Can balance detection paradigm be experimentally
  Bradley et al., J. of Neurophysiology 1987; Jacobson et al., 1975

Thanks

Alberto Romagnoni

Jérôme Ribot

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J.Ribot*, A. Romagnoni*, C.Milleret, D. Bennequin, J. Touboul,
bioRxiv doi: 10.1101/009308. Submitted to Cerebral Cortex

A. Romagnoni*, J.Ribot*,D. Bennequin, J. Touboul,
arxiv:1409.4927[q-bio.NC]. In Revision in Plos Comp. Biol.